Research plan

Takashi Sato

I will investigate **the symmetry from the geometrical viewpoint**. More concretely, I will tackle the following problems.

Prob. 1. Does the parallelogram in Figure 1 hold for other Lie types? If so, what are the corresponding objects for Hessenberg varieties of other type.

Prob. 2. GKM theory (explained later) works well for spaces corresponding flag varieties. What does GKM theory say for the relation between Hessenberg varieties and their twins?

Prob. 3. What are the geometrical objects corresponding to non-unicellular LLT polynomials?

Before going into detail aims, I introduce GKM theory. If a space has a good torus action, its equivariant cohomology ring (the ordinary cohomology ring of its Borel construction) is completely determined by the fixed point set and one dimensional orbits. This theory is called GKM theory. The construction of the twin of a Hessenberg variety is closely related to the Borel construction, and the twin itself has a good torus action.

I have concrete aims for the moment for each problems.

Aim 1. At first, I will give the correspondence of type C. Since there are Hessenberg varieties of any Lie type, next step is to construct the twin. Then calculate the representation of its cohomology, find the symmetric functions, and obtain its combinatorial description.

Aim 2. Develop GKM theory from the viewpoint of invertibility of the tori acting on a Hessenberg variety and its twin. In particular, consider some fiber bundles corresponding to them and give an explanation of an inverting phenomenon.

Aim 3. Investigate vertical-strip LLT polynomials. Then give a generalization of twins for other Lie types.

These aims are related to each other. I will partially achieve the goals, and then apply the results to other aims (see Figure 2). To develop GKM theory, I have to consider suitable class of spaces and I will obtain examples from other problems.

Figure 2 : relations among aims and way to research