Research results

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The following number [·]... correspond to the one in the list on research achievement.

•Analysis of solutions to dissipative nonlinear Schrödinger equations ([1], [3], [4], [7], [8], [11]):

We clarify the existence of a critical exponent of nonlinearities for dividing an asymptotic behavior of solutions to dissipative nonlinear Schrödinger equations. This exponent is the threshold that the mass (L^2-norm) of solutions decays or not and this one corresponds to the Fujita-exponent which is appeared in the nonlinear heat equations. The exponent also corresponds to the Barab-Ozawa critical exponent which decide the scattering situation on nonlinear Schrödinger equations under the mass conserved setting. In [4], we show the mass of solutions does not decay when the nonlinear power surpass the critical one while the mass decays if the nonlinear power is less or equal to the critical power. In this situation we call the power as subcritical or critical exponent. We also show in [1],[3],[7] that a relation between spatial regularity of solutions and a time decay rate of its mass and we lead to almost optimal decay rate when the solution have spatial analyticity. The following table implies a differential order and the time decay rate of solutions to dissipative nonlinear Schrödinger equations in one space dimension with the critical exponent. Here we introduce the table as following:

Table 1: The relation between regularity and L^2-decay

spatial regularity	$u \in H^k(\mathbb{R})(k \geq 1)$	$u \in G^s_v(\mathbb{R})$	$u \in G^1_v(\mathbb{R}) = C^\omega$
L^2-decay rate	$(\log t)^{-\frac{1}{2}+\frac{1}{2(2k+1)}}$	$(\log t)^{-\frac{1}{2}}(\log\log t)^{\frac{s}{2}}$	$(\log t)^{-\frac{1}{2}}(\log\log t)^{\frac{1}{2}}$

where $H^k(\mathbb{R})$ denotes the usual Sobolev space and $G^s_v(\mathbb{R})$ stands for the Gevrey class based on the Lebesgue L^2 space with differential index $s \geq 1$. The table 1 implies if solutions belong to $G^s_v(\mathbb{R})$ which ensures that solutions can be infinitely differentiable, then its decay rate shows almost optimal ([1]). We also showed that the upper L^2-decay estimate obtained in [3] is indeed optimal for solutions in the analytic class C_ω ([11]). In [11], we obtained special solutions which have the L^2-lower decay estimate with the same order of previous one by applying the pseudo conformal transformation associated with a kind of symmetry of the nonlinear Schrödinger equaiton.

•On the optimal mass decay of dissipative solutions ([5], [6], [10]):

It is difficult to construct smooth solutions for nonlinear Schrödinger equations with subcritical nonlinearity because a lower power nonlinearity does not regular. We consider a final state problem and construct solutions whose top term is given by the solution to the related ordinary differential equation which decays as $(\log t)^{-\frac{1}{2}}$ for $t \to \infty$.