Research Plan

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I will do researches on the following subjects.

Enumerative geometry I have been studying integrable hierarchies related to Gromov-Witten invariants in the last several years. A main target of this research since the start-up is to understand the Givental theory. A main outcome of the Givental theory is an operator formula of all-genus Gromov-Witten invariants. This formula is reminiscent of the operator formula for τ-functions of the KP and Toda hierarchies and, in some special cases, known to be related indeed to integrable hierarchies and topological recursion. On the other hand, this formula was originally proposed by A. Givental from the point of view of quantization of Frobenius manifolds and sympletic geometry. I wish to take that point of view this time to reconsider the operator formula.

KP and Toda hierarchies of B, C and D types Variations of the KP hierarchy, roughly classified into the B, C and D types, were introduced in the early 1980's just after the KP hierarchy (which amounts to the A type) was devised. Applications as well as foundations of these variants have been studied for years. For example, these variants, like the ordinary KP and Toda hierarchies, were applied to random matrix models and orthogonal polynomials. Recently, the KP hierarchy of the B type has been attracting attention in the context of enumerative geometry, in particular, Hurwitz numbers of the Riemann sphere. Interesting attempts, e.g. a Lax form in terms of pseudo-differential operators in two variables, were proposed therein. On the other hand, similar variants of the Toda hierarchy are also known since the 1980's. Recently, a completely new type of Toda hierarchy was discovered by A. Zabrodin and I. Krichever. The status of these variants are not fully elucidated.

Topological string theory and asymptotic analysis Recently, M. Alim et al. reported an approach to topological string theory on a resolved conifold (a special toric Calabi-Yau threefold) from Borel summation and exact WKB analysis. This seems to be an attempt from physicist's point of view to reconsider T. Bridgeland's Riemann-Hilbert problem for the BPS structure of Donaldson-Thomas invariants. These researches may be placed along the line of the work of R. Kashaev et al. on quantum dilogarithmic functions, and related to the work of K. Iwaki, T. Koike and Y. Takei on the τ-function of hypergeometric equations as well. Bearing these researches in mind, I will try to consider a generalization to the so called strip geometry and to incorporate the concept of isomonodromic deformations.

Grassmann manifolds and boundary measurement Arround 2006, A. Postnikov constructed a cellular decomposition of the non-negative part of a real Grassmann manifold employing the notion of boundary measurement. The boundary measurement is a mapping that sends a weighted graph to a point of the Grassmann manifold by a sum over paths or matchings on the graph. Postnikov also pointed out a relation of the cellular decomposition to diverse combinatorial notions. Since Postnikov's work, the boundary measurement and the related combinatorial notions have been applied to the description of scattering amplitude in four-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory. These researches provide a number of interesting issues, for which I expect a possible connection to twistor theory, quantum cohomology and hypergeometric functions as well.