# Future Research Plans

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## Black hole perturbation theory :

We have already derived the master equations for the massive vector field and the massive tensor field on static black hole spacetimes [1, 2], and by using these results, we have already constructed global solutions of the master equations for the massive vector and tensor fields on the massless topological black hole spacetime, and analyzed the instability and quasinormal modes. Therefore, we will extend this analysis to asymptotic-AdS black hole spacetimes and asymptotic-dS black hole spacetime, then we will attempt to analyze the master equations for the massive vector field and the massive tensor field. However, in this case, the master equations cannot be reduced into the hypergeometric equations, so we will seek a method to analyze them using some approximation method. As possible approximation methods, we will try matched asymptotic expansion, geometric-optic approximation, Born approximation, etc., and aim to construct a method to gain a global solution. Recently, it was shown that even in the complicated Kerr-Newman-de Sitter spacetime, the scalar field equations can be solved using the Heun function without any approximations [3]. We will examine whether the same analysis can be conducted for the master equations for massive fields.

## Dynamics of charged particles on dyonic black hole :

We have already analyzed the dynamics of charged particles in the case of the dyonic Reissner-Nordström black hole spacetime, and we derived the conditions that restrict the energy, charge, and mass of particles falling in an extremal black hole and the conditions that make extremal black hole over extremal. We will examine whether the same analysis can be performed not only for static spacetime but also for the case of a rotating dyonic-Kerr-Newman black hole spacetime. In addition, in the previous study [4], the ergo region, where the Penrose process can occur, has already been derived for the dyonic-Kerr-Newman black hole spacetime, therefore, as an extended analysis, we investigate the collisional Penrose process on the same spacetime. Unlike the general Penrose process, the collisional Penrose process is interesting because it is expected to be related to the BSW effect, in which Kerr spacetime is expected to play a role as a particle accelerator.

[1] K. Ueda and A. Ishibashi, Phys. Rev. D **97**, 124050 (2018).

[2] V, Cardoso, T. Igata, A. Ishibashi, and K. Ueda, Phys. Rev. D **100**, 044013 (2019).

[3] H. Motohashi and S. Noda, Progress of Theoretical and Experimental Physics 2021, 083E03 (2021).

[4] C. Dyson, D. Pereñiguez, arXiv:gr-qc/2306.1575 (2023).