Future research plans

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In the spacetime of Kerr black hole, there exists a hidden symmetry described by a special tensor field called the Conformal Killing-Yano (CKY) tensor. Such symmetry is believed to play an important role not only in the variable separation appearing in perturbation equations of black hole spacetimes but also in the analysis of (in)stability of spacetime. Historically, this symmetry was discovered by Walker and Penrose (1970) as a symmetry of 4-dimensional Kerr black hole. Furthermore, the introduction of CKY from a purely mathematical perspective dates back to the studies of geometers such as Kashiwada (1968) and Tachibana (1969). In the latter half of the 20th century, the interest in unified theories including gravity, such as superstring theory and supergravity theory, motivated the study of higher-dimensional black hole spacetimes. We have shown that CKY can be extended to higher-dimensional spacetimes [45]-[69]. The key result is the proof that the only spacetime allowing CKY is the Kerr-NUT-(Anti) de Sitter black hole spacetime [51][52].

Applying such research to geometry, it is possible to induce Einstein metrics on compact manifolds using the analytical continuation of black hole spacetimes allowing CKY. The Einstein metric obtained in joint research with Hashimoto-Sakaguchi [35] appears very natural when viewed through CKY. There seems to be a deep connection between Lorentzian geometry and Riemannian geometry. Page metric, toric Sasaki-Einstein metrics, and Einstein metrics on spherical bundles with torus action can also be constructed from higher-dimensional black hole spacetimes. An overview of these results has already been published in review papers [59] and [72][73], and further developments are expected through a deeper geometric or physical understanding.

Recent research on black holes has seen remarkable progress, including the discovery of gravitational waves (2015) and the capturing of supermassive black holes at the center of galaxies (2019). The perspective on black holes has shifted significantly from being a mathematical concept as exact solutions of the Einstein equations to being real entities. In such a period, it is valuable to reconsider classical theoretical studies of black holes using modern approaches. In the past 1-2 years, we have attempted to reconstruct the identity of operators known as Wald's four-tuple (1978) from the perspective of CKY on the most fundamental and important Kerr black hole, and we plan to continue this research. Wald's four-tuple provides a unified view for field equations on Kerr black hole and plays an important role in various calculations in astrophysics, such as gravitational wave analysis.

There is a problem in fluid dynamics concerning the motion of interfaces between fluids with different viscosities. For instance, when a low-viscosity fluid is injected into a high-viscosity fluid, the motion of the interface is described by the Laplace equation with "moving boundary conditions." In this case, highly complex geometric patterns (fractal structures) appear on the interface. The formation of such unstable patterns on moving boundaries is a common phenomenon in many scientific and technological fields. We aim to relativistically extend and study this problem here.