## Summary of research achievements

Yukinori Yasui

The doctoral dissertation focuses on the phenomenological model of elementary particles based on exceptional groups. It demonstrates that the spectrum of quarks and leptons, which is consistent with observations, can be derived from a supersymmetric nonlinear sigma model based on exceptional groups  $E_{\ell}$  ( $\ell$ =6,7,8). Furthermore, using the index theorem, it classified homogeneous spaces of exceptional groups without anomalies [2]-[6]. Subsequently, research was conducted on perturbation theory in superstring theory and soliton solutions (BPS solutions) in supergravity theory. To make supersymmetry manifest and to develop the perturbation theory, we attempted to extend the theory of ordinary Riemann surfaces to super Riemann surfaces [10]-[15]. From around 2000, the research gradually transitioned to Einstein's gravity theory. In particular, research was conducted on Ricci-flat Einstein metrics with special holonomy groups. The ability to describe soliton solutions in superstring theory and supergravity theory using such metrics has been a major motivation. A notable collaboration during this period was with Hiroaki Kanno (Nagoya University) [31][32]. We found a new Ricci-flat metric with exceptional holonomy group Spin(7).

Through Maldacena's AdS/CFT correspondence, it became possible to interpret solutions to the Einstein equations including cosmological terms as soliton solutions, not just Ricci-flat ones. In a collaborative research effort with Yoshitake Hashimoto (Osaka City University Mathematics) and Makoto Sakaguchi (Osaka City University COE Researcher), we found new compact Einstein manifolds using black hole spacetime [35]. This study extended the 4-dimensional Page metric (1979) to 5 dimensions, demonstrating that an infinite number of Einstein metrics can be induced from AdS black hole metrics. Furthermore, in the lectures held at Shizuoka Prefectural University and Nara Women's University in 2006, we provided an overview of research on the geometry of Kerr black holes and Sasaki-Einstein metrics [36]-[44].

In this context, understanding black holes became a major goal to specifically construct Einstein manifolds. While the Page metric is famous in geometry as the first example of an inhomogeneous Einstein metric, it originally embodied Stephen Hawking's idea of the universe arising from quantum tunneling. This led to starting research on black holes from a perspective different from experts in cosmology and gravity. In papers [51][52], we classified higher-dimensional black hole spacetimes with the symmetry of conformal Killing-Yano tensor fields, and our work was selected as a highlight paper for 2008/2009 in the gravitational research journal Classical and Quantum Gravity. Additionally, we were invited to contribute to a special issue on higher-dimensional black holes edited by Maeda-Shiromizu -Tanaka , where we reviewed our series of black hole research [45]-[60]. In the research task "AdS Correspondence and GIT Stability" (led by Akito Futaki) , we aimed to provide a mathematical foundation for AdS/CFT correspondence from Sasaki manifolds.