Solid arrows in the figure are results obtained in my previous works. Dashed arrows indicate future areas of study.

In my previous works, general type deformed Webster algebras $W^{\mathfrak{g}}$ are defined as subalgebras of the Khovanov-Lauda-Rouquier algebra. Since the Khovanov-Lauda-Rouquier algebra has a p-DG structure, the p-DG structure is considered for the algebra $W^{\mathfrak{g}}$. Based on the general deformed Webster algebra $W^{\mathfrak{g}}$, I will work on a construction of homological link invariants which refine the quantum invariants of links obtained from quantum groups $U_q(\mathfrak{g})$ and their representations, and also work on a construction of homological invariants of three dimensional manifolds.

Research Plan

This research will be conducted in accordance with the plan of Grant-in-Aid for Scientific Research/Area No.:22K03318.

(1) On the symmetric product $S^k(\mathbb{C}^n \otimes \mathbb{C}^m)$, we have a left $U_q(\mathfrak{sl}_n)$ action and a right $U_q(\mathfrak{gl}_m)$ action such that these actions commute. So, we have the representation

$$\gamma_m^{\mathfrak{sl}_n} : U_q(\mathfrak{gl}_m) \to \bigoplus_{\sum_{\alpha=1}^m i_\alpha = k, \ \sum_{\alpha=1}^m j_\alpha = k} \mathrm{Hom}_{U_q(\mathfrak{sl}_n)}(S^{i_1} \otimes \cdots \otimes S^{i_m}, S^{j_1} \otimes \cdots \otimes S^{j_m}).$$

It is expected that there exists a categorification of this representation on a bimodule category of the deformed Webster algebra $W^{A_{n-1}}$. I will continue to work on this challenge this year as I did last year.

(2) We can introduce the p-DG structure on the algebra $W^{\mathfrak{g}}$ defined as a subalgebra of Khovanov-Lauda-Rouqier algebra since Khovanov-Lauda-Rouqier algebras have a p-DG structure. Using the p-DG structure, I will work on a categorification of representations of quantum groups at roots of unity. I'm planning to discuss this topic with Khovavov, Lauda, Sussan, and Qi in US.

(3) The above (1) and (2) are studies on a categorification of structures appearing in symmetric tensor products. I expects that a similar categorification can be constructed in the case of anti-symmetric tensor products using the category of matrix factorizations HMF. Last year, I showed that the p-DG structure can be introduced in the category of matrix factorizations HMF. I will continue to work on this topic this year.