Research plan

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In 2003, G. Perelman proved Thurston's Geometrization Conjecture. It was proved that 3-dimensional manifolds have a unique geometric structure. Hyperbolic structures are most important among these geometric structures. It is also useful to consider hyperbolic structures when studying the properties of 3-dimensional manifolds.

If two hyperbolic 3-manifolds or orbifolds M_1, M_2 have homeomorphic finite-sheeted covering spaces, we say M_1 and M_2 are **commensurable**. "commensurability" is an equivalence relation.

The following problem is considered.

Problem 1 *For given two hyperbolic manifolds (orbifolds), determine these are commensurable or not.*

If two hyperbolic manifolds (orbifolds) are "arithmetic", this problem is solved easily. We can assume these two hyperbolic manifolds (orbifolds) are non-arithmetic.

For non-arithmetic hyperbolic 3-manifold M, there exists an orbifold $C(M)$, which is called commensurator orbifold of M. It is proved that $C(M)$ has the minimal volume among the commensurability class of M. Coomensurator is a **complete commensurability class invariant**.

For a cusped hyperbolic manifold, we can determine the commensurator of it by using Epstein-Penner decomposition. Thus, Problem 1 can be rewritten as follows..

Problem 2 *For a given compact non-arithmetic hyperbolic 3-manifold M, determine the commensurator of it.*

I have partially solved this problem. I decided the commensurators of orbifolds corresponds to the Coxeter groups of prisms and Löbell polyhedra. It is expected commensurators can be obtained for other Coxeter groups in the same way. I will generalize this method for compact hyperbolic manifolds. I want to aim for a complete solution to Problem 2

By solving this problem, the following applications are expected.

(1) In general, to show whether a hyperbolic 3-manifold is arithmetic or not, we use algebraic methods. This calculations are not easy.

I decided the commensurators of Coxeter groups by using geometrical methods. By using this geometirical method,iIt is expected to decide arithmeticity for some hyperbolic manifolds easily.

(2) The smallest and second smallest hyperbolic 3-orbifolds of volume have been determined. These orbifolds are arithmetic. It is not known the smallest non-arithmetic hyperbolic 3-orbifold of volume. The commensurator orbifolds have small volumes. By

calculating many examples, it seems to be useful for determining the minimal volume of non-arithmetic hyperbolic 3-orbfold.