Summary of research results to date
（Numbers correspond to the attached paper list）

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1 Epstein-Penner decompositions

Epstein-Penner decomposition is an ideal polyhedral decomposition for a cusped hyperbolic 3-manifold. I have studied about the property of Epstein-Penner decomposition for cusped hyperbolic 3-manifolds which contain incompressible thrice punctured spheres ([5], [7]).

By decompsing Epstein-Peneer decomposition, we proved partial result about ideal convex triangulation of cusped hyperbolic 3-manifold ([9], [10]).

2 Determination of commensurability

If two hyperbolic 3-manifolds or orbifolds M_1, M_2 have homeomorphic finite-sheeted covering spaces, we say M_1 and M_2 are **commensurable**. “commensurability” is an equivalence relation.

We costructed an arbitrary number of hyperbolic 3-manifolds which can not be distinguished commensurability by cusp parameter, and the ratio of cusp volume and that of manifold but are mutually incommensurable ([4], [8]). In [3], we show the incommensurability for three cubic Coxeter groups by calculating the commensurators of them.

In [13], for cusped hyperbolic 3-manifolds or orbifolds M_1, M_2, we show that if $0 < \mathrm{vol}(M_1) - \mathrm{vol}(M_2) < 0.252725\cdots$, M_1 and M_2 are incommensurable. By using this result, it can be seen that the set of manifolds which are obtained by Dehn filling of a cupsd manifold M, contains infinitely many commensurablity classes.

We also showed that closed hyperbolic 3-manifolds V2050(4,1) and V3404(1,3) are incommensurable by using the ratio of volumes. By generalizing this, Ryoya Kai (Osaka City Univ.) checked incommensurability of 4 million pairs in 2019.

3 Related research

Let $C(M)$ be the commensurator of a non-arithmetic hyperbolic manifold M. It is proved that $C(M)$ is the minimal hyperbolic manifold of volume in the commensurability class of M. However, few examples of computation of commensurators of compact hyperbolic manifolds (or orbifolds) are known. In [11], we calculated the commnesurators for cocompact Coxeter groups.

Let Symm(M) be the symmetry grourp of M. If $M/\mathrm{Symm}(M) \neq C(M)$, we say M has a hidden symmetry. I constructed hyperbolic 3-manifolds which have hidden symmetries. ([1], [12]).