Research Result

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Among recent research topics, 2d/4d correspondence and tensor models are described below.

• 2d/4d(5d) correspondence and some limits

The 2d/4d correspondence states the equivalence between the conformal block in 2d conformal field theory and the instanton partition function in 4d $su(n)$ supersymmetric gauge theory.

On the gauge theory side, the number of matters is $N_f = 2n$ in the original 2d/4d correspondence. By taking the mass infinity limit, the degrees of freedom of the matters are decoupled and the theory goes to that with $N_f < 2n$. On the CFT side, the counterpart is known as an irregular conformal block. The β-deformed matrix model, which is equivalent to the conformal block, converts to a unitary matrix model with log-type potentials in the irregular limit. Previous studies have been mostly concerned with $n = 1$ and little attention has been paid to the irregular limit in general n, i.e., multi-matrix modelin the irregular limit. I have established the irrgular limit procedure for $N_f = 2n-1, 2n-2$. This extension suggested that the model with general n has a rich structure that does not exist at $n = 2$. As an example, I found the following flavor mass relations that restore maximum discrete symmetry based on the Dynkin diagram at $N_f = 2n-2$. I have shown that the set of parameter spaces given by this relation maximally degenerates the corresponding Seiberg-Witten curves.

In the above mentioned unitary matrix model with log-type potential ($n = 2$, $N_f = 2$, $\beta = 1$), the Painlevé II equation appears as a string equation at the appropriate double-scale limit. The solution of this equation is related to the free energy of the theory. I investigated the behavior of the unperturbed part z in this solution and obtained,

$$z \approx \exp\left(-\frac{4}{3}\frac{1}{\kappa}\right), \tag{1}$$

where κ corresponds to the coupling constant.

On the other hand, the above quantity can also be read off by a direct calculation of the matrix model, by examining the work done against the barrier of the effective potential by a single eigenvalue lifted from the sea of the filled ones. It can be shown that the result agrees with (1), including the coefficient 4/3. Although the double scale limit has degrees of freedom for some arbitrary parameters, (1) is unaffected by them and has universality.

• tensor model

The tensor model can be regarded as a generalization of rectangular matrix model to higher rank. Recently, the tensor model is receiving a lot of attention because of its relation to the low dimensional AdS/CFT and quantum gravity. However, unlike ordinary matrix models, the gauge-invariant operators in the tensor model have nontrivial structures, which make it difficult to use ordinary methods such as the Virasoro constraints.

I demonstrated the following Op/FD correspondence between the tensor models of different ranks.

$$\text{Operator (rank } r) \quad \Longleftrightarrow \quad \text{Feynman Diagram (rank } r-1)$$

Each operator in the tensor model is, therefore, labeled with the Feynman diagram. In particular, in the case of rank 3, it is extended to an Op/FD/dessin correspondence including a one-to-one correspondence with graphs called dessins. Here the dessin is a graph consisting of vertices of two colors and edges connecting them embedded on a two-dimensional surface. I succeeded in building a concrete relationship. By using this correspondence, all operators up to level 5 in the rank 3 tensor model were classified according to properties of FD and dessin, for example, the number of vertices. In addition, I have established the interpretation of the cut & join operations as diagrammatic manipulations by expressing them in the language of dessin.