Research plan (Toshiaki ADACHI)

It is one of main subjects in differential geometry to study Riemannian manifolds by investigating properties of geodesics. In this sense, it is natural to consider that a study of some wider family of curves may give more information on the underlying manifold. Such a study was done in the field of the study of submanifolds. The applicant has been studying Riemannian manifolds which admit some geometric structures by investigating families which are associated with the structures and that induce dynamical systems on unit tangent bundles. He is planning to continue this study.

Rotation factors of Kaehler and Sasakian magnetic fields Trajectories for magnetic fields on a Riemannian manifold are determined by their velocity vectors and their acceleration vectors given by Lorentz forces. In particular, for Kaehler magnetic fields which are constant multiples of the Kaehler form, since these vectors form a complex line, one may considers that trajectories do not related to totally real directions of velocity vectors. But if we study magnetic Jacobi fields obtained by variations of trajectories, we have a component which shows that a half of the force is used to the totally real direction. When we study trajectories for magnetic fields on Riemannian manifolds endowed with magnetic fields, the study of such a component is important. As a first step, we study trajectories for Sasakian magnetic fields on totally eta-umbilic real hypersurfaces in nonflat complex space forms, because we have explicit expressions of trajectories through Hopf fibrations of the ambient spaces. Decomposing trajectories into two directions, parallel and orthogonal directions to the characteristic vector fields, we study how rotation factors appear.

L-functions of Kaehler graphs The applicant considers that Kaehler graphs having principal and auxiliary edges are discrete models of Riemannian manifolds endowed with magnetic fields. For Kaehler magnetic fields, the generating operators for random walks formed by bicolored paths are stochastic adjacency operators which are compositions of adjacency operators for principal graphs and transition operators for auxiliary graphs. When a Kaehler graph is regular and its two kinds of adjacency operators are commutative, by giving an equivalence relation on the set of bicolored paths, we can define its zeta function and can show its relationship to the stochastic adjacency operator. The applicant considers that the commutativity corresponds to fiber-structures of Riemannian manifolds, and plans to study some correspondence to Sasakian magnetic fields on real hypersurfaces. Since the relationship of zeta functions and adjacency operators is known for non-regular ordinary graphs, he wishes to study zeta functions for general Kaehler graphs. Also, to study L-functions, we need group actions on graphs. For ordinary graphs, the action is the action of the fundamental group as a CW-complex. But for Kaehler graphs, we need to take quotient groups. The applicant wishes to study a geometric meaning of these groups.

Ideal boundary of a Hadamard Kaehler manifold and trajectories When sectional curvatures of a Kaehler manifold are non-positive, if the strength of a Kaehler magnetic field is smaller than the minimum of sectional curvatures then the magnetic flow is hyperbolic, and if the strength is sufficiently large then the flow is of rotation type. Therefore, we can consider that horocycle trajectories show some properties of the underlying manifold. Here, a horocycle trajectory is a trajectory which is unbounded in positive and negative directions and corresponding limit points coincide with each other. The applicant hopes that the horosphere determined by the geodesic joining the initial point of a horocycle trajectory and its limit point gives some information on the component of a complex hyperbolic space, because properties of limit points of geodesics are related with the Euclidean factor of a Hadamard manifold. Being different from the case of geodesics, if we consider lengths of trajectories, we do not have a result corresponding to the inequality of triangles. We hence say that the situation is not so easy. But the applicant wishes to make the relationship clear.