Research results　(Toshiaki ADACHI)

A closed 2-form on a Riemannian manifold is said to be a magnetic field. Under this magnetic field, we consider motions of charged particles. Without forces of magnetic fields, these trajectories of charged particles are geodesics. Therefore, we may consider that trajectories for magnetic fields are generalizations of geodesics. When the magnetic field is closely related with the geometric structure of the underlining manifold, we can consider that properties of the underling manifold and the geometric structure are reflected to properties of trajectories. The applicant hence studied Riemannian manifolds with geometric structures by investigating properties of trajectories for associated magnetic fields.

Kaehler magnetic fields As typical examples, we have Kaehler magnetic fields, which are constant multiples of the Kaehler forms on Kaehler manifolds. On complex space forms, we can express trajectories for Kaehler magnetic fields explicitly. For a complex projective space, Kaehler magnetic flows on the unit tangent bundle are conjugate to the geodesic flow, and for a complex hyperbolic space, they are classified into three conjugate classes, hyperbolic flows, horocycle flows and rotation flows([13,19]). To study general Kaehler manifolds, the applicant used a technique of comparison. Considering magnetic Jacobi fields which are obtained from variations of trajectories, he extended Rauch's comparison theorem ([24,87,98,128]). Next, he studied trajectory-harps which are formed by trajectories and associated families of geodesics, and trajectory-horns which are formed by geodesics and associated families of trajectories. Considering that they correspond to geodesic triangles, he gave comparison theorems([91,110,115,116]), and applied them to study Hadamard Kaehler manifolds. When magnetic force is smaller than sectional curvatures of the underlying manifold, he showed that the magnetic exponential map is bijective, existence of limit points of trajectories in the ideal boundary([91,110,113,119]).

Sasakian magnetic fields on Real hypersurfaces in complex space forms On a real hypersurface in a complex space form, we have an almost contact metric structure induced from the complex structure on the ambient space. Since the fundamental form associated with the almost contact metric structure is closed because it is related with the complex structure, we can define Sasakian magnetic fields as its constant multiples([81]). As Sasakian magnetic fields are not uniform, that is, their magnetic force depend on velocity vectors, it is not easy to treat them. For each totally eta-umbilic real hypersurface, considering trajectories whose velocity vectors are orthogonal to the characteristic vector field, the Legendre magnetic flows induced by them are classified into conjugate classes up to magnetic rotation factors. This result correspond to the result on magnetic flows on complex space forms([129,130]). The applicant also studied extrinsic shapes of trajectories and gave some characterizations of homogeneous real hypersurfaces of type A ([92,106,111,126,127]). On such real hypersurfaces, we have closed and open geodesics. Applying an elementary argument of number theory, he showed how lengths of geodesics and circular trajectories are distributed in the real line ([37,39,85,121]).

Kaehler graphs A graph which consists of sets of vertices and edges are usually considered as a discrete model of a Riemannian manifold, and paths are considered as correspondences of geodesics. The applicant studied Kaehler graphs whose sets of edges are divided into two subsets, sets of principal and auxiliary edges, as discrete models of Riemannian manifolds endowed with magnetic fields. He considered that paths formed by principal edges correspond to geodesics and used auxiliary edges to show the influence of magnetic fields ([77]). By taking account of properties of magnetic means for Kaehler magnetic fields studied in [56], he considered that Kaehler graphs which correspond to homogeneous manifolds are vertex transitive Kaehler graphs whose adjacency operators associated with principal and auxiliary edges are commutative. He gave some systematic ways of their constructions([100,120]), studied isospectral but non-isomorphic ones with respect to Laplacians associated with stochastic adjacency operators([101]), and investigated the relationship between the zeta function associated with bicolored paths and that of a dynamical system([4,114]). Though Kaehler graphs are non-oriented but bicolored paths have orientations because of the order of principal and auxiliary paths. Hence we have such relationship on zeta functions. When Kaehler graphs are normal, by inducing an equivalence relation on sets of bicolored paths, we can define a zeta function of Ihara type, and can show some properties([123]).