Research plan

Takanori Ayano

1. Reduction of the abelian functions

For a positive integer g, we consider a bielliptic hyperelliptic curve $\mathcal{X}$ of genus $2g$. Then there exist two hyperelliptic curves $\mathcal{X}_1$ and $\mathcal{X}_2$ of genus g such that the Jacobian variety of $\mathcal{X}$ is isogenous to the direct product of the Jacobian varieties of $\mathcal{X}_1$ and $\mathcal{X}_2$. I will express the abelian function associated with $\mathcal{X}$ in terms of the abelian functions associated with $\mathcal{X}_1$ and $\mathcal{X}_2$. This reduction formula will contribute to the visualization of solutions of differential equations in terms of the abelian functions.

2. Jacobi inversion problem for telescopic curves

A telescopic curve is a certain algebraic curve defined by $m-1$ equations in the affine space of dimension m, which can be a hyperelliptic curve and an (n, s) curve as a special case. Let V be a telescopic curve of genus g and $P_1, \ldots, P_g \in V$. For holomorphic 1-forms $\omega = {}^t(\omega_1, \ldots, \omega_g)$ on V, let

$$u = \sum_{i=1}^{g} \int_{\infty}^{P_i} \omega.$$

Let $x_1^{(i)}$ be the x_1-coordinate of P_i. If V is a hyperelliptic curve, it is well known that $\{x_1^{(i)}\}_{i=1}^g$ are characterized as the solutions of the following algebraic equation:

$$x^g - \wp_{1,1}(u)x^{g-1} - \cdots - \wp_{1,g-1}(u)x - \wp_{1,g}(u) = 0, \tag{1}$$

where $\wp_{1,i}$ is the abelian function defined by the logarithmic derivative of the sigma function associated with V. This result is generalized to the case of (n, s) curves in [1]. I will extend this result to the case of telescopic curves, i.e., I will give an algebraic equation in the form (1) which characterizes $\{x_1^{(i)}\}_{i=1}^g$.

3. Meromorphic functions on the subvarieties of the Jacobian variety

For the hyperelliptic curves of genus $g = 2, 3$, I derived the partial differential equations integrable by the meromorphic functions that satisfy $2g$ periodicity conditions on the zero set of the sigma functions, which is a joint work with Buchstaber. These partial differential equations are two parametric deformations of the KdV equation. The zero set of the sigma function associated with a hyperelliptic curve of genus g is equal to the image of $g-1$ points on the curve by the Abel-Jacobi map. For a hyperelliptic curve of genus g, I will derive partial differential equations integrable by the meromorphic functions that satisfy $2g$ periodicity conditions on the image of $k(< g)$ points on the curve by the Abel-Jacobi map. I think that these partial differential equations will be $2g - 2k$ parametric deformations of the KdV equation.

参考文献

[1] J. Bernatska, D. Leykin, Solution of the Jacobi inversion problem on non-hyperelliptic curves, Lett. Math. Phys. **113**, 110, (2023).