Previous research

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Klein generalized the elliptic sigma function to the multivariable sigma functions associated with hyperelliptic curves. Buchstaber, Enolski, and Leykin generalized the hyperelliptic sigma functions to a plane curve called (n, s) curve. I extended the sigma functions for the (n, s) curves to telescopic curves.

A hyperelliptic abelian function of genus g is a meromorphic function on $\mathbb{C}^g$ that satisfies $2g$ periodicity conditions on $\mathbb{C}^g$. Buchstaber, Enolski, and Leykin proved that the hyperelliptic abelian functions satisfy the KdV equation. For hyperelliptic curves of genus 3, I constructed the theory of the meromorphic functions that satisfy 6 periodicity conditions on the zero set of the sigma functions and derived the partial differential equations integrable by these functions, which is a joint work with Buchstaber. These partial differential equations are two parametric deformations of the KdV equation.

Enolski and Salerno expressed the abelian functions associated with a bielliptic hyperelliptic curve of genus 2 in terms of the Jacobi elliptic functions. I expressed the same abelian functions in terms of the Weierstrass elliptic functions, which is a joint work with Buchstaber. Further, I expressed the abelian functions associated with a bielliptic hyperelliptic curve of genus 3 in terms of the Weierstrass elliptic functions and hyperelliptic functions of genus 2. In physics and biology, it is necessary to compute the values of hyperelliptic functions. By using the reduction formula, it is easier to compute the values of the hyperelliptic functions. Our results will contribute to these fields.

In [2], it is proved that the coefficients of the power series expansion of the sigma function for an (n, s) curve are included in the ring generated by the coefficients of the defining equation of the curve over $\mathbb{Q}$. I extended this result to telescopic curves, which is a joint work with Nakayashiki. In [3], it is proved that the coefficients of the power series expansion of the sigma function for an (n, s) curve are included in the ring generated by the coefficients of the defining equation of the curve and $1/2$ over $\mathbb{Z}$. I extended this result to telescopic curves.

In the paper in 1903, Baker defined the abelian functions associated with a real hyperelliptic curve, derived a fundamental formula on differential relations between these functions, and gave differential relations for genus 3 explicitly. In [1], it is proved that the abelian function of a real hyperelliptic curve satisfies the KP equation for genus 3. I gave differential relations of the abelian functions of a real hyperelliptic curve explicitly and prove that the abelian function of the real hyperelliptic curve satisfies the KP equation for any genus, which is a joint work with Buchstaber.

参考文献

[1] S. Matsutani, Hyperelliptic solutions of KdV and KP equations: re-evaluation of Baker's study on hyperelliptic sigma functions, J. Phys. A: Math. Gen. **34** (2001), 4721–4732.

[2] A. Nakayashiki, On Algebraic Expressions of Sigma Functions for (n, s) Curves, Asian J. Math. **14** (2010), 175–212.

[3] Y. Ônishi, Arithmetical Power Series Expansion of the Sigma Function for a Plane Curve, Proc. Edinburgh Math. Soc. **61** (2018), 995–1022.