The plans on my research

Cheng Qing-Ming

Since curvature flows are very powerful tools for studying geometric structures of Riemannian manifolds, ones have been studying geometric structures on curvature flows from different directions. Many important achievements are obtained. Research on critical points of functionals plays an very important role in the study on singularities of curvature flows. We consider to study geometric structure on critical points of functionals. Especially, as the plans on our research, we propose to study the following two relative subjects.

1. We try to study classifications of complete self-shrinkers as critical points of Gaussian area functionals and to study λ-hypersurfaces as critical points of Gaussian area functionals for preserving the weighted volume variations. In particular, (1). We try to generalize the Alexandrov type theorem of 2-dimensional self-shrinkers, which was proved by Brendle, to higher dimensional case.(2). We will extend the results on Wiggly conjecture and the plane domain conjecture of Ilmanen which were resolved by Brendle in two directions. Firstly, for n-dimensional complete self-shrinker, we try to propose conjectures of Wiggly and the plane domain conjecture of Ilmanen. Then, we will study them. Secondly, we will try to express the conjecture of Wiggly type and the plane domain conjecture of Ilmanen type for proper complete λ-surfaces and to study them. Furthermore, we also consider whether there exist counter-examples.

2. We try to study complete minimal hypersurfaces as critical points of area functionals and to study complete hypersurfaces with constant mean curvature as critical points of area functionals for preserving the volume variations. As a generalization of Bernstein theorem, do Carmo and Peng, Fischer-Colbrie and Schoen, Pogorelov proved that the plane is the only complete two-sided stable minimal surfaces in Euclidean space R^3. On the other hand, for $n>6$, Bombieri, De Giorgi and Giusti were able to construct n-dimensional complete two-sided stable minimal hypersurfaces in Euclidean space R^{n+1} which are not hyperplane. The following conjecture is very famous:

Conjecture. For $2<n<7$, an n-dimensional complete two-sided stable minimal hypersurface in Euclidean space R^{n+1} is a hyperplane.

It is our purpose to study the above conjecture. We also study complete δ-stable minimal hypersurfaces, which is a generalization of stable minimal Hypersurfaces. Stable hypersurfaces with constant mean curvature are also studied.