Future Research

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As before, we will conduct research to develop existing theorems into theorems in CAT space. We investigate fixed point approximation theorems in Hilbert space, Banach space, and Hadamard space (complete CAT(0) space), and consider what theorems can be developed in CAT(κ) space. In many cases, the recurrence formula that approximates the fixed point can be used as is. On the other hand, we will need to devise reasonable assumptions for the mapping. It is well known that in Hilbert space, a projection or resolve to a nonempty closed convex set becomes a nonexpansive mapping. On the other hand, although counterparts can be defined in CAT(1) space, they have poor properties and are not nonexpansive. When researching the development of existing theorems in CAT(1) space, the basic approach is to first conduct research with the assumption that the mapping that approximates a fixed point is nonexpansive (although there are few applications). If the expected result can be proven, it can be generalized to the assumption of "strongly quasi-nonextended and Δ-demiclosed" (this assumption includes nonexpansive mappings and resolvents in CAT(1) space are included). Recently, we have also been paying attention to properties such as "asymptotically nonexpansive" and "I-nonexpansive" for a given mapping I, and the direction of generalization after setting the assumption "non-extension" and obtaining results. We plan to conduct several such events. Specifically, the goals are as follows:

[1] presents a theorem that simultaneously approximates the minimization point and fixed point when one convex function and one mapping are given in ACT(1) space. On the other hand, in [2] and [3], when multiple convex functions and maps are given in Hadamard space, the recurrence formula used in [1] is changed to the method of [2] and [3]. We have obtained results that are developed into a common fixed point approximation theorem, which is expected naturally. Therefore, we are conducting research to develop the results in [2] and [3] into the CAT(1) spatial fixed point approximation theorem. At the moment, I have been conducting research that relies on inequality evaluation and formula transformation, which I have already conducted by examining previous research. From now on, we plan to proceed with our research using the methods described in [4], [5], [6], and [7]. In [4] and [5], it is assumed that the mapping under consideration has an "asymptotically" property, which is very similar to the current research. In addition, [6] proposes a method for simultaneously approximating the minimization point of a convex function and the fixed point of a mapping in CAT(1) space, which is also useful. [7] approximates the minimization point of one convex function and the fixed points of multiple mappings in

CAT(1) space (however, the shape of the recurrence formula currently being studied is different), and this also It's helpful. No matter which one is, it cannot be applied as is, so it is necessary to evaluate the inequality independently as appropriate. Therefore, there is a possibility that this research will be conducted in collaboration with Yasunori Kimura (Toho University) and Keisuke Shindo (National Institute of Technology, Hachinohe).

Reference

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