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Future Research Plans

Following my previous research, I aim to further investigate solitary waves in nonlinear dispersive equations from multiple perspectives, seeking to elucidate their intrinsic mathematical properties. Specifically, I plan to explore the following three research themes related to solitary waves.

1. Stability and Instability for All Frequencies and Velocities. For equations where scale invariance or Galilean invariance is broken, previous studies have primarily analyzed stability in cases where the frequency or velocity is sufficiently small or large—situations that can be reduced to simpler equations. However, in intermediate cases or for algebraic solitary waves, such perturbative methods are difficult to apply. Moving forward, I aim to develop new techniques that enable the application of abstract stability theory and further extend the theory itself.

2. Properties as Solutions to Nonlinear Elliptic Equations. Understanding the existence, uniqueness, and non-degeneracy of solutions to the nonlinear elliptic equations that govern the shape of solitary waves is essential for stability analysis. Recent studies have also highlighted the importance of detailed information on the derivatives of solitary wave families with respect to frequency or velocity, including their decay/growth rates and behavior at the origin, in stability analysis. My future research will seek to extend existing results to establish the existence, uniqueness, and non-degeneracy of solutions over a broader range. Additionally, I will conduct a rigorous analysis of the derivatives of solitary wave families using elliptic equation theory, ordinary differential equation theory, and operator theory. By applying the properties obtained from this analysis to stability studies, I aim to advance research in this direction.

3. Asymptotic Stability and Strong Instability. The strength of stability varies depending on parameters such as frequency and velocity, leading to phenomena such as asymptotic stability (where solutions near a solitary wave converge to it) and strong instability (where solutions near a solitary wave blow up). However, asymptotic stability and strong instability are difficult to determine using abstract theory alone and often require case-by-case analysis for each equation. As a result, these aspects remain unexplored for many equations. In future work, I plan to study asymptotic stability and strong instability in various equations, including nonlinear Schrödinger equations with potentials or point interactions. By refining and applying linear estimates and variational structures associated with solitary waves, I aim to uncover universal conditions for the occurrence of these stability and instability phenomena.