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A summary of past research achievements

My research focuses on solitary waves (solitons) in nonlinear Schrödinger equations. Solitary waves are special solutions that propagate while maintaining their shape at a constant frequency and velocity. They have been widely studied as objects of significant interest in both physics and mathematics. In particular, I am interested in how the stability and instability of solitary waves change depending on the equation, frequency, and velocity. Below, I summarize the main results obtained so far (the numbers in [·] correspond to the references in my publication list).

Strong Instability of Solitary Waves. When a potential is present or in the case of doubly power-type nonlinearities, the breakdown of scale invariance generally makes the analysis of strong instability difficult. Previous studies established strong instability for solitary waves with positive energy, but this assumption imposed a strong restriction, as it corresponded to a situation similar to the single power-type case. In contrast, in papers [3] and [4], I proved the strong instability of solitary waves under more natural and general assumptions from the perspective of scaling.

Algebraic Solitary Waves. Certain types of nonlinear Schrödinger equations admit not only exponentially decaying solitary waves but also algebraically decaying solitary waves in critical situations. In paper [6], through an analysis based on energy and scaling, I derived sufficient conditions for algebraic solitary waves to be unstable and strongly unstable. In paper [11], I successfully extended an abstract theoretical approach to algebraic solitary waves in one dimension, proving instability under more general conditions than in [6]. Furthermore, in paper [10], I demonstrated the existence of algebraic solitary waves in a system of nonlinear Schrödinger equations where Galilean invariance is broken.

Solitary Waves in Nonlinear Schrödinger Equations with Singular Potentials. For nonlinear Schrödinger equations with strongly singular potentials, fundamental properties of solitary waves remain partially unexplored. In paper [5], I established the uniqueness and non-degeneracy of positive radially symmetric solitary waves in the case of an inverse power potential. In paper [8], I investigated the two-dimensional nonlinear Schrödinger equation with point interactions, proving the local well-posedness of the initial value problem and the existence and positivity of radially symmetric solitary waves. Additionally, I obtained results on the stability and instability of solitary waves with small or large frequencies.