SUMMARY ON MY PREVIOUS RESEARCH

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The applicant has mainly studied about a branched twist spin that is a 2-knot in S^4. The branched twist spin consists of exceptional orbits and fixed points of a locally smoothly effectively circle action on S^4 and, by definition, it is invariant under the circle action. Since the circle action induces a fibration on the knot complement, the branched twist spin, except the spun knot, is fibered.

It is known by Plotnick that a fibered 2-knot is the branched twist spin if and only if its monodromy is periodic. Thus the branched twist spin is a 2-knot version of a torus knot.

Branched twist spin is also obtained from the twist spun knot and a branched covering map. Let $\tau_m(K)$ be the m twist spun knot of K and suppose that m and n are coprime. Then the n-th covering of S^4 along $\tau_m(K)$ is S^4 again and The preimage $\tau_{m,n}(K)$ of $\tau_m(K)$ by the covering map is 2-sphere. We call $\tau_{m,n}(K)$ the (m,n) branched twist spin of K.

Since the branched twist spin is determined from three parameters K, m and n, The applicant is interested in how the knot type are changed when the parameters are changed. The applicant has obtained the following result about classifying branched twist spins:

Theorem 1 (F. [2])**.** Let $\tau_{m_1,n_1}(K_1)$ and $\tau_{m_2,n_2}(K_2)$ be non-trivial branched twist spins. If either the following conditions holds, then $\tau_{m_1,n_1}(K_1)$ と $\tau_{m_2,n_2}(K_2)$ are not equivalent.
(1) m_1 and m_2 are even and, $|\Delta_{K_1}(-1)| \neq |\Delta_{K_2}(-1)|$,
(2) m_1 is even, m_2 is odd and $|\Delta_{K_1}(-1)| \neq 1$.
Here, $\Delta_K(t)$ is the alexander polynomial of K.

Theorem 1 is obtained by comparing the Alexander ideals of $\tau_{m_1,n_1}(K_1)$ and $\tau_{m_2,n_2}(K_2)$. The same result also obtained by counting the number of irreducible $\mathrm{SL}_2(\mathbb{C})$-metabelian representation of $G(K^{m,n})$.

Theorem 2 (F. [2])**.** Suppose that m is odd. Then $K^{m,n}$ and $K^{m,m+n}$ are not equivalent.

From the fact that a fibered 2-knot with odd monodromy is not equivalent to its dual by Gluck twisting, Theorem 2 is obtained since $K^{m,m+n}$ is obtained from $K^{m,n}$ by Gluck twisting.

Theorem 3 (F.-Ishikawa [3])**.** Let K_1 and K_2 be non-equivalent and non-torus prime knots. Suppose that m is greater than 2. If $Z(\pi_1^{orb}\mathcal{O}(K_i,m))$ is trivial, then Let m be an integer greater than 2 and $K_1^{m,n}$ と $K_2^{m,n}$ are not equivalent.

If $Z(\pi_1^{orb}\mathcal{O}(K,m))$ is trivial, then $G(K^{m,n})/Z(G(K^{m,n}))$ is isomorphic to $\pi_1^{orb}(\mathcal{O}(K,m))$. Thus, for such a case, $\pi_1^{orb}(\mathcal{O}(K,m))$ is invariant of $\tau_{m,n}(K)$. Theorem 3 is obtained by using Mostow rigidity for orbifolds, and Takeuchi's result for a sufficiently large knot in orbifolds.

Since twist spining, that is defined by Zeeman, can be repeated, we obtain the 3-knot $\tau_{m_2}(\tau_{m_1}(K))$ from the 1-knot K by twist spinning twice. The same statement as in Theorem 3 holds for $\tau_{m_2}(\tau_{m_1}(K_1))$ and $\tau_{m_2}(\tau_{m_1}(K_2))$ with $m = \gcd(m_1, m_2)$

References

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