(2) Research plan

"Deepening Hamiltonian fluid and magnetohydrodynamics through Nambu mechanics and its application to the stability of a compressible shear layer"

Nambu bracket is a magic wand that lies at the core of non-canonical Hamiltonian formalisms. For 2020-22, I chaired the OCAMI symposiums entitled *"Nambu mechanics for linking space-time topology with formation of micro-macro magneto-vortical structure"*, which expanded possibility of Nambu mechanics for enlightening space-time topology. Using the deepened Nambu mechanics[115)] as a lever, I will tackle the puzzles that hang over the stability of a compressible shear layer.

1. Deepening non-canonical Hamiltonian structure for fluid mechanics and magneto-hydrodynamics (MHD) by Nambu brackets

Non-canonical Poisson brackets for fluid dynamics and MHD take complicated forms. Its structure becomes clear if we derive the Nambu-bracket representation by a hint that the Casimir invariants of MHD are exhausted by four, including the cross helicity. Moreover, Arnold's theorem which characterizes stationary solutions of incompressible Euler flows and gives the wave energy formula, can be extended to compressible MHD.

i) The Nambu bracket representation is sought for the extended MHD (Hall effect, electron inertial effect) and characterize the stationary solution.

ii) By extending Arnold's theorem, I calculate the wave energy on a steady flow and the mean flow (zonal flow) induced by wave interactions. The isomagnetovortical disturbance is derived using the Nambu bracket, and thereby a general formulas for the wave energy and mean flow are derived.

iii) Complementary to the top-down approach based on Nambu mechanics is a bottom-up method that relies on the Friedman-Rotenberg (FR) equation for the Lagrangian displacement of MHD.

2. Instability of a vortex sheet and a shear layer in a compressible fluid

After a half century since the Concorde, development of supersonic aircrafts has resumed, and vortex dynamics in the supersonic regime has attracted attention. In the incompressible approximation, a vortex sheet, which is the limit of a shear layer with a thickness of 0, always causes Kelvin-Helmholtz (KH) instability, amplifying two-dimensional wavy deformation, and the amplification rate is proportional to the velocity difference ΔU. Compressibility suppresses it when ΔU becomes even larger and the Mach number $M=\Delta U/c$ (c: sound speed) exceeds $\sqrt{8}$. This surprising result is limited to two dimensions, and larger ΔU is required to suppress three-dimensional instability. In addition, another instability resides inside the shear layer. Armed with **the wave energy formula for compressible fluids**, I will unravel the tangled threads in stability of a compressible shear layer.

i) "Linear stability analysis of a shear layer of finite thickness" will be carried out, and instabilities localized within the layer will be characterized using the Hamiltonian spectral theory. Calculation of the energy of waves standing on the shear layer holds the key.

ii) The effects of gravity and surface tension on compressible KH instability will be investigated. Preliminary calculations show that gravity destabilizes a stabilized supersonic velocity discontinuity, even when a light fluid is above a heavy fluid. So is the surface tension. It is a puzzle that the gravity force and the surface tension, which are restoring forces, act to destabilize the interface stabilized by compressibility. This puzzle is to be resolved by appealing to the energetics.