RESEARCH PLAN

MASAAKI FURUSAWA

I plan to work on the following topics.

Applications of Böcherer's conjecture

In a joint work (Compositio Math. **160** (2024)) with Kazuki Morimoto, we proved a generalization of Böcherer's conjecture to the case when the character on the torus is not necessarily trivial. This theorem is naturally regarded as a generalization to $\mathrm{GSp}\,(4)$ of the Waldspurger formula in the $\mathrm{GL}\,(2)$ case, which has found many applications. We expect our formula to find various applications and we ourselves would like to find them in light of the $\mathrm{GL}\,(2)$ case.

Matrix argument Kloosterman sum

The classical Kloosterman sum is an exponential sum which have found many important application in number theory. There exist various generalizations of the classical Kloosterman sum. One of them is the one which appears in the Fourier coefficients of the Poincaré series with respect to the Fourier coefficients along the unipotent radical of the Siegel parabolic subgroup of Siegel cusp forms. This Kloosterman sum has symmetric matrices as its variables and has been studied by Christian, Kitaoka and others. In our relative trace formula approach to Böcherer's conjecture, we naturally encountered with such Kloosterman sums in the degree two case and obtained some explicit formulas. I would like to find applications of the formula by considering the interpretation of the Kloosterman sum in terms of the geometry behind it.

Periods of algebraic automorphic forms

In a joint work with Morimoto, we proved the algebraicity of the critical values of the L-function for $\mathrm{SO}\,(V) \times \mathrm{GL}\,(2)$ where V is a definite quadratic space. There the Deligne period is given by a power of the Petersson norm of the $\mathrm{GL}\,(2)$ cusp form and no information of the $\mathrm{SO}\,(V)$ is reflected. Though it is possible to compute the exact values of periods of automorphic forms on anisotropic algebraic groups, the meaning of them is not clear except for a few cases. I would like to find some examples of the periods which are related to special values of automorphic L-functions and their number theoretical applications.