Preceding results

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I concern functional inequalities such as Hardy-type or uncertainty principle inequalities for concstained vector fields. The Hardy inequality, first found by G. H. Hardy for one-dimensional scalar fields, was developed by J. Leray to that for higher-dimensional vector fields, in the context of Navier-Stokes equations. Today it is applied to many directions. A matter of special interest for me is how the best constants in the functional inequalities change from the original ones when the test functions are vector-valued and are subject to differential constraint such as curl-free or solenoidal condition. This problem originates from the one suggested by Costin-Maz'ya in 2007, who derived a new best constant in the Hardy-Leray inequality for axisymmetric solenoidal fields. In view of their results, I tried to remove the axisymmetry condition or extend such results to other types of inequalities. On this situation, I have ever obtained the following results:

Sharp Hardy-Leray inequality for solenoidal fields

For the case of three-dimensional solenoidal fields, I found in a joint work [9] with Prof. F. Takahashi that the same best constant as Costin-Maz'ya can be obtained by restricting the axisymmetry condition on only the azimuthal components of the test fields. Furthermore, in [2] and [7] I extended this result to the case of general-dimensional solenoidal fields without any symmetry assumption at all, and incidentally found in [4] a simpler expression for the best constant in view of the dependence on the weight exponent.

Sharp Rellich-Leray inequality for solenoidal fields

As a second-order version of Hardy inequality, I derived in the paper [1] the new best constants in Rellich inequality with weights for axisymmetric solenoidal fields, by giving a rigorous characterization of axisymmetric vector fields. Moreover, this result was extended in [3] to the case of general solenoidal fields, and the non-attainability of the best constants was also shown.

Sharp Hardy-Leray and Rellich-Leray inequalities for curl-free fields

Following from the two-dimensional case of Costin-Maz'ya's result, in a joint work [11] with Prof. F. Takahashi, I derived sharp constants in weighted Hardy and Rellich inequalities for higher-dimensional curl-free vector fields. We also proved in [12] the non-attainability of the best constants found in the former work, by estimating remainder terms. With regard to the critical case, we verified in [10] that the best constant in the logarithmic weighted Hardy inequality for two-dimensional curl-free fields coincides with the one for unconstrained fields.

Sharp uncertainty principle inequality for solenoidal fields

Heisenberg's uncertainty principle inequality is well known for scalar fields together with the explicit best constant. In [5, 6] I derived the new best constant in the same inequality for solenoidal vector fields and also found that it can be attained by a class of solenoidal fields. Moreover, the profiles of the extremal functions was clarified.