② Research plan **(1)** More specific goals are the following. Consider a Ricci flow $(g(t))_{t\in[0,T)}$ on a closed four dimensional (smooth) manifold whose scalar curvature is bounded on the interval $[0, T)$ $(T > 0)$. For such a Ricci flow, **I will specify what one of the following situations will occur**.

(1-a) There is no finite-time singularities. (i.e., There is no $0 < T < \infty$ such that $|\mathrm{Rm}(g(t))|_{g(t)} \to +\infty$ as $t \to T$).

(1-b) There is a finite time $0 < T < \infty$ where some singularities suggested in [1, 4] (or [9]) occur. (If this case does occur, I will also try to classify that types of singularities in terms of [5].)

(2) More specific goals are the following.

(2-c) Giving a characterization of the lower bound of the total scalar curvature on a closed manifold via some sort of convexity of the space. (cf. [8]). Firstly, investigating a generalization of Green's theorem [6].

(2-d) In my paper [10], I gave a definition of <u>lower</u> bounds of total scalar curvatures for $W^{1,p}$ $(p > n)$ metrics on a closed n-manifold. Investigating those properties and relations between other weak notions of lower bounds of total scalar curvatures (e.g., in the distributional sense).

(2-e) Giving a definition of the total scalar curvature <u>upper</u> bound of a C^0 metric on a closed manifold using the Yamabe flow, and investigating its properties. (This problem is caused by my own paper [11].)

③ Longer term planning As a longer-term plan, I would like to use geometric analysis methods to tackle the various problems listed in "Gromov's four lectures [7]". Until now, various gauge theoretic methods have been successful in 4-dimensional differential topology. On the other hand, in [3], the validity of a geometric analysis method using the Ricci flow is suggested. Therefore I would like to approach some problems around 4-dimensional differential topology in such a direction in the future.

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