Research statement (Shota Hamanaka)

I'm interested in the shape of a manifold endowed with a Riemannian metric structure, and the topology of some distinctive subspaces of the space of all Riemannian metrics, e.g., the space of positive scalar curvature metircs. The main method that I'm using is geometric analysis, especially Geometric flows (e.g., Ricci flows). I'm also interested in Ricci flows themselves and study the behavior of them. I'll concisely explain my past and current studies (papers) as follows.

- S. Hamanaka, Decompositions of the space of Riemannian metrics on a compact manifold with boundary, Calc. Var. Partial differential Equations 60, 1-24 (2021).**(Peer-reviewed)**

  In this paper, I extended Koiso's decomposition theorem (on a closed manifold) to one on a compact manifold with boundary.

- S. Hamanaka, Non-Einstein relative Yamabe metrics, Kodai Math. J. 44, 265-272 (2021).**(Peer-reviewed)**

  In this paper, I gave a sufficient condition for a constant scalar curvature metric with minimal boundary to be a relative Yamabe metric. Here, a relative Yamabe metric is the one corresponding to a solution of the Yamabe problem with minimal boundary condition.

- S. Hamanaka, Ricci flow with bounded curvature integrals, Pacific J. Math. 314, 283-309 (2021).**(Peer-reviewed)**

  In this paper, I proved that if a Ricci flow on a closed manifold satisfies a condition characterized as some integral forms, then as the time approaches the first finite singular time, the metric converges to a Riemannian metric on the manifold except for finitely many singular points. I also showed that such a flow can be extended over the singular time as an orbifold Ricci flow.

- S. Hamanaka, Type of finite time singularities of the Ricci flow with bounded scalar curvature, arXiv:2105.08250 (2021).**(Not peer-reviewed)**

  In this paper, I showed that the shape of finite-time singularities of a Ricci flow on a closed manifold with bounded scalar curvature are restricted in some sense.

- S. Hamanaka, Limit theorems for the total scalar curvature (old version: C^0, C^1-limit theorems for total scalar curvatures), arXiv:2208.01865v14 (2022).**(Submitted)**

  In this paper, I particulary proved the following: Let $\mathcal{M}$ be the space of all Riemannian metrics on a closed n-manifold M ($n \geq 3$). Then, for any nonnegative continuous function σ and constant κ, the space

  $$g \in \mathcal{M} \quad \int_M R(g)\, d\mathrm{vol}_g \geq \kappa,\ R(g) \geq \sigma$$

  is closed in $\mathcal{M}$ with respect to $W^{1,p}$ ($p > n$)-topology.
  In the same paper, I also proved that the lower bound of the weighted total scalar curvature

  $$\int_M R(g) e^{-f}\, d\mathrm{vol}_g$$

  is preserved under certain convergences of metrics and weight functions. Here, f is some weight function on the manifold M.

- S. Hamanaka, Upper bound preservation of the total scalar curvature in a conformal class, arXiv:2301.05444v5 (2023).**(Submitted)**

  In this paper, I particulary proved the following: Let $g_0 \in \mathcal{M}$. Then, for any continuous function σ and constant κ, when the conformal class $[g_0]$ of g_0 is Yamabe positive or nonpositive respectively,

  $$g \in [g_0] \quad \int_M R(g)\, d\mathrm{vol}_g \leq \kappa,\ R(g) \geq \sigma \quad , \quad g \in [g_0] \quad \int_M R(g)\, d\mathrm{vol}_g \leq \kappa$$

  are closed in $\mathcal{M}$ with respect to C^0-topology respectively.

- S. Hamanaka, Notes on scalar curvature lower bounds of steady gradient Ricci solitons, arXiv:2409.00583v2 (2024). **(Not peer-reviewed)**

  In this paper, the author investigated scalar curvature lower bounds of steady gradient Ricci solitons via μ-bubbles, which is a generalization of minimal hypersurfaces.