I plan to study rational structures of representations.

Loewy gave **a classification scheme of finite dimensional real irreducible representations of a group** G and determined **the division algebras of their endomorphisms**. According to him, there is a natural bijection between the set of isomorphism classes of finite dimensional real irreducible representations of G and that of complex conjugacy classes of isomorphism classes of finite dimensional complex irreducible representations of G. He also showed that for a finite dimensional (self-conjugate) complex irreducible representation of G, the division algebra of endomorphisms of the real irreducible representation corresponding to V is determined by the so-called **index**. It is a sign **determined by** V.

Then Tits gave a similar classification result for connected reductive algebraic groups G over a field F of characteristic zero. That is, he gave a bijection between the set of isomorphism classes of irreducible representations of G and that of Galois conjugacy classes of isomorphism classes of irreducible representations of $\bar{F} \otimes_F G$, where $\bar{F}$ is an algebraic closure of F. For a self-conjugate irreducible representation V of $\bar{F} \otimes_F G$, he gave a **2-cocycle which determine (the opposite algebra to) the division algebra of endomorphisms of the irreducible representation of** G **corresponding to** V. To be precise, this cocycle was introduced by Borel–Tits as an obstruction class to existence of an F-form of V. We call it the **Borel–Tits cocycle** of V. This is determined by V. For general V, we can reduce ourselves to the self-conjugate case by taking the base change of G to the field of rationality of V. Note that this result is **valid for general affine group schemes**. Then we can regard it as a generalization of Loewy's result explained above (for G finite). The **Borel–Tits cocycles generalize the indices**.

Papers 1 and 5 are subsequent works to them. In fact, all these works of Loewy, Tits, and mine are obtained from **generic discussions on Galois descent**. I plan to introduce a comprehensive framework to them, and give a similar result in that setting. Namely, I **classify simple objects over a field** F **by the** Γ**-conjugacy classes of those over a separable closure** F^{sep}, where Γ is the absolute Galois group of F. To be precise, J will give a natural bijection between the set of isomorphism classes of a certain f-linear abelian category and the quotient of the set of isomorphism classes of a certain F^{sep}-linear abelian category by Γ. Roughly speaking, this implies that theories over F are understood in terms of those over F^{sep}. This bijection is meaningful since theories over F^{sep} tend to be easier than those over F for rationality issues. For instance, Tits classified irreducible representations of G in terms of dominant weights by using the highest weight theory over $\bar{F}$ in his work explained above. I also **construct the obstruction class to existence of** F**-forms** as a straightforward generalization of the Borel–Tits cocycles. Analyzing that for a (self-conjugate) simple object X over F^{sep}, I will determine a **minimal field** F' **of definition of** X (i.e., a minimal finite separable extension F'/F in F^{sep}, where X admits an F'-form).

Then I will confirm that this abstract formalism is applicable to $(\mathfrak{g}, K)$-modules. This leads us to a **classification result of irreducible** $(\mathfrak{g}, K)$**-modules over a field** F **of characteristic zero**. I also apply this theory to cohomologically induced modules to obtain **their minimal fields of definition**.