Future research plan

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I will study the following in analysis for random functions:

(1) Ogawa integrability

In [5], we showed that the Ogawa integral $\int_0^t X(s)\, d_\varphi B_s$ with respect to a Brownian motion B of a certain Skorokhod integral process X converges for each $t \in [0, L]$. In the causal case, [7] indicates that this convergence is uniform in $t \in [0, L]$, and therefore X satisfies the condition called regular integrability. Besides, [8] gave the unique existence of the stochastic integral equation by the regular integrable random function. So, I investigate the regular integrability of X in the general case. Also, I study the Ogawa integrability of the random function which is integrable with respect to the Nualart-Pardoux-Stratonovich integral (NPS integral for short) presented in [6, Theorem 7.3] and the Ogawa integrability in the case that the space $[0, L]$ of time parameter t is generalized.

(2) Riemann approximation of the stochastic integral

Introducing the regularized Riemann sum $R_\Delta(X; Y)$ for random functions X and Y and a weighted partition Δ of $[0, T]$, I constructed the stochastic integral $I_\alpha(X; Y) = \int X d_\alpha Y$ based on R_Δ indexed by a measurable function α from $[0, T]$ to $[0, 1]$ with the following two property ([3]):

(a) When Y is a Brownian motion B and $\alpha = \frac{1}{2}$, I_α approximates the Ogawa integral regarding regular CONSs and NPS integral.

(b) When Y is a continuous semi-martingale and $\alpha = \frac{1}{2}$, I_α approximates the Fisk-Stratonovich integral with respect to Y.

When Y is a Brownian motion, [7] shows the fundamental fact on the Ogawa integral that ”every Brownian quasi-martingale is φ-integrable” is equivalent to ”φ is regular” for a CONS φ which defines the Ogawa integral. On the analogy of this fact, when Y is a continuous quasi-martingale, I gave a necessary and sufficient condition for a weighted partition Δ to I_α exists for every quasi-martingale.

The Ogawa integral is defined with respect to a Brownian motion, while the integral I_α given in [3] is defined with respect to a quasi-martingale and includes as a special case the Ogawa integral regarding regular CONSs in the sense in (a). So, I want to answer the question whether the same fact holds for the integral I_α with respect to a (continuous or discontinuous) quasi-martingale as on the integrability and application given in [9] for the Ogawa integral.

(3) Stochastic differentiability, identification of random functions from the SFCs

The class of random functions with the quadratic variation and derivative $\frac{d}{dV}$ with respect to a random function V with the quadratic variation, described in (3) in ”Summary of research results so far”,

$$Q(V) = \left\{ X : \text{random function} \;\middle|\; [X],\ \frac{dX}{dV} \text{ exists and } \frac{d[X]}{d[V]} = \left|\frac{dX}{dV}\right|^2 \right\}$$

forms a vector space ([4]). Next, we work on examining whether $Q(V)$ forms a ring and giving a larger linear space than $Q(V)$ as a class of random functions with the quadratic variations. Besides, given random functions V and W with quadratic variation, we are to give the calculation formula for quadratic variation or derivative of $X + Y$, XY when $X \in Q(V)$ and $Y \in Q(W)$. On the other hand, giving the derivation formulas based on the Ogawa integral and integral I_α in (2), we verify that these integral processes belong to $Q(V)$. Further, we attempt to apply this study to the identification problem of random functions from the SFCs.

References

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