

## Future Research Plan

January 30, 2025 Jinko Kanno

After finishing graduate school in Japan, I started studying topological graph theory and continued learning graph theory in the rest of my working career. I have realized that the concept of topology has been foreign to the most graph theorists. Graph Minors are popular in topological graph theory such as Robertson-Seymour graph minor publications; however, Graph Immersions are studied by a few groups of researchers. Since I am one of them, I feel responsible to explain the concept of graph immersion to more people. I have submitted a paper titled "Splitter theorems for 4-regular planar graphs," but it was returned because the length of the paper did not match the journal. Without publishing this paper, my job is not complete.

In Knot Theory, I am working with colleagues on Kauffman's bracket polynomials; for a given link diagram  $D$ , Kauffman defined a unique 3-variable polynomial  $\langle D \rangle (A, B, d) \in Q[A, B, d]$  by using inductive rules. To make them be a link invariant, we have to normalize with self writhe of  $D$ ; let  $S(D)$  be the normalized 3-variable polynomial. However, the polynomial  $S(D)$  itself is not a link invariant yet. We want to make it be a link invariant by adding some relations among  $A, B$ , and  $d$ . Let  $I$  be an ideal of  $Q[A, B, d]$  and consider the quotient ring  $Q[A, B, d]/I$ . We are looking for a special ideal  $I$  such that the quotient map from  $Q[A, B, d]$  carries  $S(D)$  to a class  $[S(D)] \in Q[A, B, d]/I$ , which is a link invariant.