Research outline

Takuya Katayama

I have been working on geometric group theory. Geometric group theory is an area devoted to studying algebraic and geometric properties of groups endowed with the structure of a metric space, given by the so-called word metric. Especially, I have studied injective homomorphisms from right-angled Artin groups (RAAGs) into various groups and their applications by using topology and graph theory.

(1) On injective homomorphisms from RAAGs

For every RAAG and knot in S^3, I decided whether the RAAG is embedded in the fundamental group of the knot complement. The knot complement can be decomposed into a union of small 3-manifolds that are easy to study by JSJ decomposition. I obtained the above result by using JSJ decomposition and reducing the problem on general knot complements to the problems on specific 3-manifolds. I also studied injective homomorphisms between RAAGs. I proved that we can construct a full injective graph homomorphism between the defining graphs from an injective group homomorphism from RAAG of the complement graph of a linear forest into the RAAG of a graph. This result led me to give necessary and sufficient conditions for embedding RAAGs of the complement graphs of line graphs into the mapping class group of orientable surfaces.

(2) On mapping class groups of surfaces

(Joint work with Erika Kuno) We gave necessary and sufficient conditions for embedding pure braid groups, finite index subgroups of the Artin's braid group, into the mapping class groups of orientable surfaces by using results in (1). It is known that the almost all orientation-double-coverings induce injective homomorphisms $(*)$ between the mapping class groups of surfaces. On the other hand, Koberda gave a method for embedding RAAGs into the mapping class groups of orientable surfaces. By combining these results, we gave a method for embedding RAAGs into the mapping class groups of non-orientable surfaces. Furthermore, we proved that the injective homomorphisms $(*)$ are quasi-isometric embeddings by using theory of semi-hyperbolic groups. We also gave an alternative proof of the Hamenstädt's theorem that states inclusion maps between surfaces induce quasi-isometric embeddings of the mapping class groups.

(3) On curve complexs of surfaces

Recently, I have studied curve complexes with the aim of understanding Masur-Minsky theory for mapping class groups more precisely. In 2023, with Erika Kuno, I improved Hempel's inequality regarding the distance of curve complexes and calculated the constant of bounded geodesic image theorem. In 2024, we calculated hyperbolic constants for the curve graphs of closed surfaces including non-orientable ones. We are currently preparing a paper on this topic.