Future research plans Kotaro Kawai

I want to construct an invariant by G_2-dDT connections. I expect to obtain more interesting results in the case of G_2-dDT connections. The reason is: (i) (More precisely,) G_2-dDT connections correspond to "graphical" calibrated submanifolds, and the singular set seems to be more tractable. (ii) From [15, 16], the moduli space of G_2-dDT connections seems to have better properties than calibrated submanifolds and G_2-instantons. (iii) Significant research has been done on dHYM connections, with deeper results than calibrated submanifolds.

[Plan 1] Bubbling analysis of minimal connections

To construct invariants, it is necessary to study the compactification of the moduli space and investigate its properties in detail. To do this, we first need to know how the bubbles occur. We consider analogues of the Yang-Mills case, which are similar to minimal connections and we understand how bubbles occur.

The key points in the Yang–Mills case are (i) Price's monotonicity theorem and (ii) Uhlenbeck-Nakajima's ε regularity theorem. So far, I have tried to show these analogues. I have shown a certain analogous result to (i) in paper [22], but the result might be made a little stronger. (In fact, we were able to show a stronger statement for G_2-dDT connections.) We construct examples by the symmetry of Lie groups or the gluing, and consider to what extent the results can be strengthened based on it. In addition, I will try to find the relation between the monotonicity theorem and scaling of the "volume" V (in some sense), as in the Yang–Mills case.

For the analogy to (ii), it is necessary to show that the "energy density" (integrand) v of V satisfies the "Bochner-type inequality". (That is, v should be a subsolution of an elliptic operator of divergence form.) Since v is much more complicated than the energy density of the Yang–Mills functional, it will be more difficult to prove such an inequality. However, I showed in paper [22] a formula analogous to the Weitzenböck formula, which was important in the proof of the Yang–Mills case, and found that terms with the highest derivative would be handled well. The low-order derivative terms still seems to be complicated, but I expect that they can be estimated well with some more technical ingenuity.

[Plan 2] (Morse-)Floer homology using G_2-dDT connections

From "My achievements (Ⅲ)", there is an observation for G_2, Spin(7)-dDT connections analogous to that of instanton Floer homology (IFH) for 3-manifolds. As a further development of [Plan 1], we might be able to construct a Floer homology using G_2-dDT connections. As analogies to IFH, I will first consider whether (a) the deformation complex of Spin(7)-dDT connections on the cylinder is Fredholm and its (relative Morse) index is given "nicely", and (b) the moduli space is smooth by the "holonomy perturbation".

Then, referring to other Floer homology theories, I would like to consider the following. (1) Is it possible to compactify the moduli space of Spin(7)-dDT connections on the cylinder? (2) If the Floer homology can be constructed, to what extent does it depend on the geometric structures and perturbation? (3) Can we define a higher-order product structure such as the A_∞ structure for this Floer homology?

[Plan 3] Submanifolds determined as the "mirror" of connections

We have considered transferring the notion of submanifolds to the connection side via the "real Fourier–Mukai transform" so far. Recently, in collaboration with T. Pacini of University of Turin, we have been trying to introduce a new notion of submanifolds from the connection side conversely.

So far, we have found that if a G_2-manifold has a certain fiber bundle structure (this is a canonical assumption in the context of mirror symmetry), then "mirror" of a G_2-instanton can be defined, which we call the "Fueter-type section". This can be considered as a variant of the Fueter section that appears in the bubbling analysis of G_2-instantons. We have also found that it minimizes certain functionals, as in the cases of calibrated submanifolds, G_2-instantons, and G_2-dDT connections.

Therefore, as in the past, we will first investigate whether the "Fueter-type section" has similar properties to G_2-instantons. More specifically, we will consider (i) What is the deformation theory? (ii) Are there any functional whose critical points are Fueter-type sections? (iii) Does the gradient flow equation have a good geometric interpretation? (iv) How do bubbles occur? In particular, (iv) is known for the standard Fueter section, so we would like to start based on it.