My achievements

Kotaro Kawai

(I) Reserach of calibrated submanifolds

I have mainly studied the properties of explicit calibrated submanifolds. Explicit examples are very important because they provide local models of singularities and are used for the desingularization by the gluing construction. However, calibrated submanifolds are defined by nonlinear PDEs, which makes the explicit construction very difficult. In [1], I constructed examples of special Lagrangian submanifolds in a toric Calabi-Yau manifold by applying the moment map technique on $\mathbb{C}^n$ by Joyce. In [2, 9, 19], I classified homogeneous coassociative submanifolds and gave many cohomogeneity one examples in an asymptotically conical G_2-manifold. In particular, I could find examples with conical singularities and their desingularizations.

In [4, 6, 18, 7], I studied the infinitesimal (first order) deformations and the second order deformations of explicit homogeneous Cayley cones for the analysis of singularities. By the representation theory and geometric considerations, I clarified the local structures of moduli spaces of some of these cones.

(II) Research of the topology and the moduli space of $G_2, \mathrm{Spin}(7)$-manifolds

In [8, 10, 20], I studied $G_2, \mathrm{Spin}(7)$-manifolds using an algebraic structure called the Frölicher Nijenhuis bracket in collaboration with H. V. Lê and L. Schwachhöfer. In [12], together with H. V. Lê, L. Schwachhöfer and D. Fiorenza, I further developed and generalized this. From the consideration of general graded differential algebras, we found new topological obstructions for a manifold to admit a metric with non-negative Ricci curvature. (Especially, these are new obstructions for a manifold to admit $G_2, \mathrm{Spin}(7)$-structures.) In [11], I found the common properties of moduli spaces of various geometric structures. As an application, I investigated the difference in the structure of the metric completions when the canonical metric in the space of the G_2 structures or the Ebin metric in the space of the Riemannian metrics is conformally transformed.

(III) Similarity of calibrated submanifolds and their "mirrors"

We can consider dHYM, $G_2, \mathrm{Spin}(7)$-dDT connections as an analogue of calibrated submanifolds or HYM, G_2, Spin(7)-instantons, but it is necessary to confirm whether the similarity holds rigorously. In [13–16], I showed that many similarities in the moduli spaces actually hold in collaboration with H. Yamamoto.

In [13, 15], we show that the moduli space of dHYM, $G_2, \mathrm{Spin}(7)$-dDT connections is a smooth manifold with the canonical orientation under the perturbation of geometric structures, and the connected components are tori.

By the "real Fourier Mukai transform" on the torus bundles, Lee-Leung defined Spin(7)-dDT connections, but there were problems such as incompatibility with other geometries. We did this carefully again, and proposed an alternative definition of Spin(7)-dDT connections which seems more appropriate in [14].

It is known that there exists a functional whose critical points are G_2-dDT connections. My paper [17] states that under certain conditions, "the gradient flow equation can be interpreted as the defining equation of the Spin(7)-dDT connection on the cylinder." This is analogous to an observation in instanton Floer homology for 3-manifolds. This provides a new link between 3, 4-manifold theory and G_2, Spin(7)-geometry.

The "volume" for connections V is defined as a "mirror" of the volume of the submanifold. (In physics, it is called Dirac-Born-Infeld (DBI) action.) There is fundamental but important equalities in $G_2, \mathrm{Spin}(7)$ geometry, called Cayley, associator equality. My paper [16] shows that the "mirror" of these equalities holds, and using this, we show the following.

(a) $G_2, \mathrm{Spin}(7)$-dDT connections minimize the "volume" V, and its value is topological. (This corresponds to the fact that each calibrated submanifold is volume minimizing in its homology class, and the volume is topological.) (b) Any $G_2, \mathrm{Spin}(7)$-dDT connection on flat bundles must be flat. (c) The structure of the moduli space is determined when the holonomy group of a G_2- or Spin(7)-manifold reduces to smaller subgroups.

Relatedly, we showed the short-time existence and uniqueness of the negative gradient flow of the "volume" V (which can be considered as the "mirror" of mean curvature flow).

(III') Properties of "mirrors" of minimal submanifolds (minimal connections)

Analogous to minimal submanifolds, critical points of the "volume" V for connections are called minimal connections. (From [16], $G_2, \mathrm{Spin}(7)$-dDT connections minimize V, so they are minimal connections.) In [22], we showed that minimal connections are "mirrors" of minimal submanifolds and are analogous to the Yang–Mills connections. In addition, I showed a monotonicity theorem for minimal connections, which is roughly analogous to the Price's monotonicity theorem for Yang–Mills connections. As an application, I obtained the vanishing theorem (Liouville-type theorem) on $\mathbb{R}^{2m+1}$.