$\boxed{\text{Future studies}}$ We expect the following must develop.

- **Generalizations of special generic maps, related examples and classifications in specific situations.** We have obtained theorems on non-existence of special generic maps on manifolds such as projective spaces (Articles 4.1-2、4.4-7): special generic maps may not cover wide classes of manifolds. We have previously considered some generalizations of such maps. Images of these maps are codimension 0 immersed manifolds and the preimages of points in the interiors are closed manifolds: originally the preimages are diffeomorphic to the unit spheres and those of points in the boundaries are single points (Articles 4.8-9). We have obtained non-trivial examples. Finding new general properties, examples and classifications are important problems to attack. We expect that these maps can be tools in projective spaces, toric manifolds and naturally generalized manifolds in transformation group theory, for example.

- **Constructing real algebraic functions and maps.** Nash, followed by Tognoli etc., says that we can know the existence of such maps in considerable cases. It is difficult to have examples with information on precise global structures, important polynomials etc. For example, we need to apply existing methods and new ones from real algebraic geometry.

- **On classifications of Morse(-Bott) functions.** Classifications of Morse(-Bott) functions on given manifolds (or classes of manifolds) are of important problems, being applicable to geometry of manifolds. These functions are also fundamental and interesting objects in geometry and singularity theory. Michalak, followed by Gelbukh, recently, classified Morse-Bott functions on closed surfaces via Reeb graphs. We have succeeded in a kind of classifications of Morse functions on fundamental manifolds, represented as connected sums of the products of the circle and the 2-dimensional sphere, and so-called Lens spaces (Article 4.13). Articles 4.15-16 are related studies. We will consider higher dimensional cases, for example.

- **Our studies in real algebraic and real analytic geometry?** In "Constructing real algebraic functions and maps", to find meanings of our study in such fields is difficult and important. Now, even low dimensional real algebraic sets and manifolds are hard to construct, classify etc. Our studies are, as we think, considering higher dimensional versions of lower dimensional cases naturally or respecting our characteristic thoughts.

- In constructing real algebraic maps as presented, we have constructed maps onto regions surrounded by circles, respecting the canonical projections of the unit spheres. Related to this, we are interested in arrangements of circles and have launched singularity-theoretic and combinatorial studies on the arrangements. We have defined several meaning classes and studied shapes of the regions by natural graphs the regions collapse to (Article 4.14). We will study further: we will find new meaning classes and investigate fundamental properties such as properties on shapes.