**Masahide Manabe**

# Research Accomplishments

I am a theoretical physicist, who is interested in mathematical physics, in particular, topological string theory, matrix models and supersymmetric gauge theories. By using various techniques in string theory and gauge theories, I have worked on exploring non-trivial relations between seemingly different mathematical (especially enumerative geometric) objects and developing their computational techniques. The following is an overview of some of my works. The reference numbers are the ones in "List of Publications".

**Topological recursions.** The CEO TR by Chekhov-Eynard-Orantin can be considered, in general, for a 1D algebraic variety $\Sigma = \{ (x,y) \in \mathbb{C}^2 \,|\, A(x,y) = 0 \}$ which is called a spectral curve, and defines multilinear meromorphic differentials $\omega_h^{(g)}(z_1,\dots,z_h)$, $h \geq 1$, $g \geq 0$ on $\Sigma$, from the Liouville one-form $\omega_1^{(0)}(z) = ydx$ and a bidifferential $\omega_2^{(0)}(z_1,z_2)$ that are (classically) defined for $\Sigma$, where $z$ is a local coordinate on $\Sigma$: $x = x(z), y = y(z)$. The CEO TR has its origin in the loop equations of matrix models, and they have applications to the models related to 2D gravity. I will summarize my works relevant to it:

- For a class of local toric Calabi-Yau 3-folds (CY3s), the differentials $\omega_h^{(g)}(z_1,\dots,z_h)$ are identified with generating functions of open Gromov-Witten invariants of genus $g$ for certain Lagrangian submanifolds. Furthermore, around 2010, via geometric engineering, it was conjectured that they also give correlators for a surface operator in 4D $\mathcal{N} = 2$ $SU(N)$ supersymmetric gauge theories. We discussed it for $SU(2)$ gauge theories in detail [2,4].
- Around 2009, by Dijkgraaf and Fuji, it was proposed an embedding of the volume conjecture in 3D $SL(2,\mathbb{C})$ Chern-Simons gauge theories into the topological string theory. Here, the moduli space of flat connections in a Chern-Simons gauge theory is described by an algebraic variety called an $SL(2,\mathbb{C})$-character variety, and for a knot complement in $S^3$ the character variety gives a 1D spectral curve. We conjectured that for character varieties of knots, the CEO TR gives a (large color) asymptotic expansion of colored Jones polynomials of knots, and checked it by non-trivial examples [3].
- A class of matrix models has free field realizations in 2D and is deeply related to 2D conformal field theories (CFTs). For instance, for such matrix models, the CEO TR is derived as loop equations, and they are shown to be equivalent to the Virasoro constraints in 2D CFT. We showed that an infinite family of quantum (spectral) curves, associated with a spectral curve in hermitian matrix models, can be explicitly constructed by the CEO TR and identified with Belavin-Polyakov-Zamolodchikov differential equations for an infinite family of Virasoro singular vectors in 2D CFT [8]. We also discussed a supersymmetric analogue of quantum curves [9,13].
- The Weil-Petersson volumes for moduli spaces of bordered Riemann surfaces satisfy the Mirzakhani's recursion. In 2017, as a generalization of the recursion, Andersen-Borot-Orantin proposed the ABO topological recursion, and relations to Virasoro constraints and the CEO TR are also discussed. By applying them to the 2D $(2,p)$ minimal gravity with an odd integer $p$ and its supersymmetric analogue, we discussed recursive structures of "volumes" for them in detail [21].

**Exact partition functions.** Triggered by the work by Pestun in 2007 about the exact computation of partition functions and correlators in $\mathcal{N} = 2$ supersymmetric gauge theories on $S^4$, the supersymmetric localization technique used there has been applied to obtain exact results in supersymmetric quantum field theories in various dimensions and background geometries. I will summarize my works achieved using such exact results:

- In 2012, Jockers-Kumar-Lapan-Morrison-Romo conjectured that, when a gauged linear sigma model (GLSM) describes a CY manifold, the GLSM partition function on $S^2$ gives the exact Kähler potential on quantum Kähler moduli space of the CY manifold. We applied their method to CY4s and conjectured an exact formula of the Kähler potential on the quantum Kähler moduli space of CY4s [5].
- Using the A-twisted GLSM partition functions, we achieved the followings: − Computation of the B-model Yukawa couplings on local toric CYs (a proposal of systematic introduction of twisted masses to remedy the subtlety due to the non-compactness of local toric CYs) [15]; − Computation of the Givental $I$-functions for some of determinantal CYs via "factorization" of the partition functions of A-twisted GLSMs [16]; − Construction of the off-shell Bethe wavefunctions in $\mathfrak{su}(N)$ XXX (XXZ) spin-chain, via 2D (3D) Bethe/Gauge correspondence, as orbifold-type codimension-2 defects (generalization of earlier works in the case of $\mathfrak{su}(2)$) [17].
- Using the A-twisted partition functions (twisted indices) of 3D $\mathcal{N} = 2$ gauge theories on $S^2 \times S^1$, we constructed and proposed an abelian gauge theory, that we called "knot-gauge theory", whose K-theoretic vortex partition functions give the colored Jones polynomials of knots in $S^3$ [20].