

(2) Study proposal

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(i) Aim of the study

The aim of the study is to understand algebro-geometric aspects of algebraic complex $K3$ surfaces, which we simply call $K3$ surfaces. The study of geometry of $K3$ surfaces is deeply involved in classical areas such as singularity theory, automorphism groups, and algebraic curve theory, and in modern area of mathematical physics. For our purposes, we are expected to analyze certain lattices associated to $K3$ surfaces together with symplectic automorphism actions on them, and topological characteristics of them. We also regard it as important to focus on classical examples of $K3$ surfaces and their relation with algebraic curves. We consider the following problems:

Problems

1. $K3$ surfaces admitting symplectic automorphisms.
2. Moduli of maps from a $K3$ surface to S^1 .
3. Double covering structure of weighted $K3$ hypersurfaces.
4. Weierstrass semigroups of pointed curves and $K3$ surfaces.

(ii) Study methods

Problem 1 Let X be a $K3$ surface admitting a finite automorphism group G acting symplectically on X , and L the lattice generated by all the classes of irreducible components of the exceptional divisor of the resolution of the singularities in X/G . Does the lattice L admit a unique primitive sublattice of the $K3$ lattice containing L ?

Problem 2 We are interested in studying the moduli space of maps from a $K3$ surface to S^1 as a higher-dimensional generalization of the loop group. We expect that the topological aspects of $K3$ surfaces would help to investigate these objects.

Problem 3 The aim of this part is to settle down the question that which weighted $K3$ surfaces admit a structure of double sextic, and what is the polarization that gives the structure.

Problem 4 (A joint work with Professor Jiryo Komeda in Kanagawa Institute of Technology) We are interested in studying which Weierstrass semigroups for pointed curves are on/off a $K3$ surface, and we'd also like to give a characterisation for Weierstrass semigroups attained by an algebraic curve on a $K3$ surface that is the triple covering of a rational surface.

(iii) Aspects

We expect as a consequence that we can understand algebraic and topological properties of geometry of $K3$ surfaces. We hope to extend our area of study for non-projective $K3$ surfaces : automorphism groups, behaviour of the periods.