(2) Abstract of results Makiko Mase

The aim of the study is to understand the algebro-geometric structure of algebraic $K3$ surfaces, which we simply call $K3$ surfaces, admitting symplectic automorphisms.

Let X be a $K3$ surface, and we denote the nowhere-vanishing holomorphic 2-form on X by ω_X. An automorphism g on X is said to be symplectic if the induced action g^* on the 2-form ω_X is trivial.

Abelian finite symplectic automorphism groups on $K3$ surfaces are classified into fourteen isomorphic classes by V.V.Nikulin in 1980. In 1988, S.Mukai shows that any finite symplectic automorphism groups on $K3$ surfaces are contained in the Mathieu group of order 23. Finally, all finite symplectic automorphism groups on $K3$ surfaces are classified into 81 isomorphic classes, for each of which, its commutator sugroup, and other important invariants of them are determined by Z.Xiao in 1996.

Let G be a finite group that consists of symplectic automorphisms on X. The action of G on X produces the quotient space X/G which contains at most simple singularities. Moreover, the space X/G is birationally equivalent to a $K3$ surface, say, Y. Indeed, Y is obtained by resolving the singularities in X/G, which provides the exceptional divisor E (on Y) that forms the union of lattices of type ADE.

Here, let us consider the lattice L_G that is generated by the classes of all the components in the exceptional divisor E.

The lattice L_G is known to be a sublattice of the $K3$ lattice Λ_{K3} that is the even unimodular lattice of signature $(3, 19)$. However, it is not necessarily true that the lattice L_G is a primitive sublattice of Λ_{K3}. If it were the case, then, we may be able to understand the $K3$ surface Y to be polarized by L_G. If not, we may set the following problem: can you determine the unique primitive sublattice $\tilde{L}_G$ in Λ_{K3} that contains L_G ?

There are studies concerning the lattice L_G and the problem for some special cases. First of all, the lattice L_G for each G is determined by Xiao in 1996. In all the cases that the finite symplectic automorphism group is abelian, Nikulin gives the affirmative answer for the problem, with explicit unique primitive sublattices in 1980. If the group is non-abelian and simple, then, U.Whitcher studies the problem and gives answers in 2011. In this case, the answers are not always affirmative.

Following the backgrounds, we are to attack the problem. In our study, we consider the problem for all remaining cases, namely, when neither G nor the abelization group $Q := G/[G, G]$ is trivial. Up to now, we have some partial answers: If Q is the cyclic group of order 2 or 3, then, we can prove that the primitive closure $\tilde{L}_G$ of L_G uniquely exists.