# Research Plan

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**Construction of dynamical Einstein system using normal almost contact metric manifolds**

**Background and Objectives：** The applicant previously collaborated with Ishihara to construct an exact solution to a four-dimensional Einstein system with a charged dust fluid and a contact magnetic field as matter, using a three-dimensional Sasakian manifold. This demonstrated the strong compatibility between contact geometry and Einstein systems. While constructing exact solutions with functional degrees of freedom is generally difficult, the geometric properties of Sasakian manifolds made it feasible. However, the complexity of the matter field, though physically reasonable, left room for mathematical refinement.

The applicant then realized that Einstein systems with NG-strings as matter could also be solved using Sasakian manifolds and confirmed this with a simple example, suggesting broader applicability. This study aims to develop a systematic method for constructing exact solutions of Einstein systems in four-dimensional dynamical and inhomogeneous spacetimes, using generalized Sasakian manifolds called normal almost contact metric manifolds, with NG-strings as matter.

**Significance：** The connection between contact geometry and general relativity has rarely been discussed. While some studies note the appearance of contact structures in certain spacetimes, there has been no clear formulation of Einstein systems using contact geometry[*1].

The applicant has focused on utilizing contact geometry in constructing Einstein systems, yielding promising results. If successful, this approach would further demonstrate contact geometry as a valuable tool in general relativity, contributing meaningful insights to mathematical relativity.

**Plan：** First, several concrete examples will be computed to identify common properties. Next, existing papers on normal almost contact metric manifolds will be studied in depth to understand how their geometric properties influence the four-dimensional spacetimes constructed from them. Then, the identified properties will be analyzed in relation to the characteristics of contact manifolds. Finally, these findings will be formulated into a systematic method for constructing Einstein systems with NG-strings as matter using normal almost contact metric manifolds.

The applicant is also collaborating with Ishihara, Koike, Furusaki, and Morisawa, combining expertise and discussions to refine the approach.

**Construction of new Einstein system in Robinson spacetimes**

**Background and Objectives：** The applicant studied contact manifolds in general relativity, constructing four-dimensional spacetimes by taking a direct product of time with a three-dimensional contact manifold, leading to exact Einstein system solutions with dust fluids, spinor fields, and electromagnetic fields. While time seemed the natural choice, another approach is to use a null line, forming Robinson spacetimes, which play a role in CR geometry and relativity. Ishihara, Koike, Furusaki, and Morisawa previously constructed Einstein systems with NG-strings in simple Robinson spacetimes. This study aims to generalize their work and establish a systematic construction method.

[*1] Kholodenko, Arkady L. Applications of Contact Geometry and Topology in Physics, World Scientific, 2013.