Research Results

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A contact manifold is an odd-dimensional manifold with a one-form called a contact form. Sasakian manifolds, a kind of contact manifold, appear in various areas of mathematical physics, notably in AdS/CFT. Another important example is the study of contact magnetic fields, where the contact form is treated as a gauge field. While contact geometry has applications in general relativity, studies focusing on contact magnetic fields are limited.

The applicant's research explores the application of contact geometry to general relativity, including two studies from 2023 – 2024: (1) The Einstein-Dirac-Maxwell system in static Sasakian spacetimes, and (2) The geometric properties of Sasakian quasi-Killing spinors on three-dimensional pseudo-Riemannian Sasakian manifolds.

① **Study on the Einstein-Dirac-Maxwell System in Static Sasakian Spacetimes**：A contact magnetic field is an electromagnetic field interpreted as a gauge field of the contact form on a Sasakian manifold. We found that when an electric current is distributed along the Reeb vector field, the resulting magnetic field satisfies the Maxwell equations. If this current comes from a charged dust fluid flowing along the Reeb vector, an exact solution for the coupled system can be constructed. In a four-dimensional static Sasakian spacetime, we obtained an exact solution to the Einstein system involving the contact magnetic field, charged dust fluid, and gravity[*1]. Since a fluid is a macroscopic system, we expected an analogous exact solution in the Einstein-Dirac-Maxwell (EDM) system, where the interacting field is a spinor instead. Given the complexity of the EDM system, constructing exact solutions is significant. We successfully found one using a Sasakian quasi-Killing spinor, a special spinor field on Sasakian manifolds. This result has been accepted for publication in PTEP[*2].

② **Geometric Properties of Sasakian Quasi-Killing Spinors on Three-Dimensional Pseudo-Riemannian Manifolds**：Sasakian quasi-Killing (SqK) spinors, introduced by Friedrich et al., generalize Killing spinors on Sasakian manifolds. While SqK spinors on three-dimensional Sasakian manifolds have been classified, their geometric and physical properties remain largely unexplored. This study investigates SqK spinors on three-dimensional pseudo-Riemannian Sasakian manifolds, revealing the following key results:

(i) The Dirac current of a special SqK spinor coincides with the Reeb vector field, a fundamental object in contact geometry. (ii) The motion of a charged particle in a contact magnetic field is described by the Dirac current of an SqK spinor, offering a new perspective. (iii) Most SqK spinors solve the Einstein-Dirac system with a cosmological constant, while special SqK spinors and contact electromagnetic fields provide exact solutions to the Einstein-Dirac-Maxwell system, emphasizing their physical significance. (iv) We obtained the first explicit expression of SqK spinors in a specific frame, which had not been given before.

These results were presented at JGRG33 and summarized in a paper[*3], currently under review for publication.

[*1] H. Ishihara, S. Matsuno, PTEP 2022.2 (2022): 023E01.

[*2] https://doi.org/10.1093/ptep/ptae098

[*3] arXiv:2308.10432v2