

Research Results --- Yasuo Matsushita  
For 2024 Academic Year  
January 4, 2024

One of the major subjects of Professor Matsushita is the differential geometry and topology of indefinite metric spaces. The main results are as follows:

The latest manuscript, submitted on Nov 23, 2024

For a compact oriented 4-manifold, it is known that the existence conditions of a field of 2-planes, a pair of an almost complex structure and an opposite almost complex structure, and a neutral metric on the manifold are equivalent to each other. We exhibit metrics on a 4-manifold admitting a field of 2-planes, which are compatible with the field of 2-planes and with two kinds of almost complex structures. It is shown that for a given 4-manifold admitting a field of 2-planes there exist ten compatible metrics, consisting of a Riemannian metric and nine neutral metrics. Depending on a 4-manifold and on a choice of a field of 2-planes, we obtain symplectic structures, almost Kähler structures, or Kähler structures, not only of Riemannian metrics, but also of neutral metrics.

1. The existence condition for a compact oriented 4-manifold to admit a neutral metric of signature  $(++--)$  are settled. In relativity theory the existence condition of a Lorentz metric on a manifold is well-known. Therefore, the problem of the existence condition for the indefinite metric of neutral signature on 4-manifolds is interesting because such a metric is the lowest dimensional example of indefinite metrics not of Lorentz type. Its existence condition is known as same as the existence of a field of 2-planes. Applying Theorem of Hirzebruch and Hopf (1951) and Donaldson's works, celebrated as the Fields Prize in 1986, the existence condition of a neutral metric on a 4-manifold is expressed in terms of the Euler characteristic and the Hirzebruch index of the manifold, which are both topological invariants. These results are cited in Donaldson's Book "The Geometry of Four-Manifolds," published by Oxford University Press, in 1990.
2. One of the other significant results is to show a counter example constructed on a Walker 8-manifold to the Goldberg conjecture, posed in 1969, which states that an almost complex structure of a compact Einstein almost Kähler manifold must be integrable, in other words, a compact Einstein almost Kähler manifold must be Kähler.
3. It is shown that the Euler characteristic and the Hirzebruch index must of a compact neutral Einstein 4-manifold must obey an inequality, which is similar to the Hitchin-Thorpe inequality for Riemannian-Einstein 4-manifolds, only except the sign.
4. Moreover, it is proved that such an existence condition of a neutral metric on a 4-manifold is equivalent to the existence condition of a pair of an almost complex structure and an opposite almost complex structure on the 4-manifold.
5. On the basis of the Enriques-Kodaira classification of compact 2-dimensional manifolds, usually called *surfaces*, it is shown that a surface admits an opposite almost complex structure if and only if the second Chern class of the surface is even.
6. Petean reported a new example of a neutral Einstein Kähler 4-manifold. A general method of constructing new such examples of neutral Einstein Kähler 4-manifolds are proposed in terms of arbitrary 2-dimensional harmonic functions.
7. Generalizing the notion of null vectors, we proposed isotropic tensors, with property of the zero squared norm of tensors. As one of the significant examples, a new example of isotropic Kähler structure is constructed on an Enle 4-manifold.

8. Recently, some substantial results are obtained on the spinor approach to neutral geometry of 4-manifolds.

The other activities: Mathematical Reviews Reviewer (from 1981). The Editor-in-Chief of JP Journal of Geometry and Topology. An Interview, with a photo, by Institute Of Physics of Great Britain was opened during several years in the IOP Home Page. A Chairperson of International Meeting in India. A Keynote Speaker in Symposium in Turkey. An External Examiner for PhD evaluation in King Saud University in Saudi Arabia. And many other activities.