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Research History

Black holes are considered an ideal setting for testing general relativity under strong gravitational fields, with their properties characterized by two parameters: mass and angular momentum. This unique nature manifests in the "ringdown" phase, where black holes "resonate." The waveform during this phase contains complex eigenfrequencies known as quasinormal modes (QNMs), which reflect the dissipative nature of black holes and have been studied in detail based on black hole perturbation theory. Moreover, QNMs are not only significant in astrophysics but are also discussed in theoretical physics fields such as quantum gravity and holography.

Mathematically, QNMs are defined as complex eigenvalues of second-order ordinary differential equations with two-point boundary conditions. Numerical methods, such as the Leaver method, have been developed to solve these problems. However, analytically understanding the global structure of QNMs in the complex frequency plane remains challenging. One approach involves constructing approximate solutions at boundaries and connecting them analytically. In this process, the QNM condition is embedded in the coefficients of the solutions and is expressed in a form analogous to the Bohr-Sommerfeld quantization condition in quantum mechanics. This method allows for the analysis of higher modes (eikonal limits) and highly damped modes of QNMs.

Over the past 40 years, significant progress has been made in "exact WKB analysis," which resums all orders of WKB perturbation. This analysis describes the Stokes phenomenon in the complex coordinate plane using Stokes curves and enables analytic continuation of solutions without approximation. Using this method, the QNM boundary condition can be rigorously expressed as a phase integral of odd-order WKB series.

To date, only limited studies have applied exact WKB analysis to black hole QNMs. A related method, the monodromy method, specializes in highly damped QNMs. However, it introduces approximations before analytic continuation, limiting its applicability. In contrast, exact WKB analysis derives the QNM condition without approximation, eliminating principal constraints on its application. This characteristic reduces the difficulty of QNM calculation to the evaluation of phase integrals. Furthermore, constructing solutions along the real axis clarifies discussions regarding black hole reflection and transmission coefficients.

Our research has developed a computational method for black hole QNMs using exact WKB analysis. Specifically, analytic continuation along the real axis from the event horizon to infinity revealed the contributions of logarithmic spiral-like behaviors of Stokes curves and branching lines originating from the horizon, which had previously been overlooked. Considering these features allowed us to reproduce known results in solvable models and Schwarzschild black holes. This novel approach provides a distinct methodology for QNM analysis and suggests broader applications in black hole physics.