Toric face rings can be considered as a generalization of Stanley-Reisner rings. That is, while Stanley-Reisner rings are connection of polynomial rings connected according to the structure of a simplicial complex, toric face rings are connection of Ehrhart rings connected according to the structure of a polytopal complex. For this reason, toric face rings have been actively studied recently. However, it seems that there has not been much research into the properties of toric face rings with special origins. As I wrote in "Previous research results", I have shown that there are toric face rings obtained from a Hibi ring. That is, Hibi rings can also be expressed as normal affine semigroup rings, the canonical module of normal affine semigroup rings has a representation by Stanley, and is expressed as an ideal of that normal affine semigroup ring. We call this ideal the canonical ideal. I have shown that the fiber cones of the canonical ideal and its inverse in the divisor group (which is a fractional ideal, and we will call it the anticanonical ideal below) are both toric face rings. This result was obtained in the study of Frobenius complexity of Hibi rings by investigating the fiber cones of the anticanonical module, which is closely related to the Frobenius complexity.

Meanwhile, I am currently studying the limit Frobenius complexity not only of toric rings, such as Hibi rings, but also of other rings. In particular, I am planning to investigate whether the limit Frobenius complexity is maximum in terms of the structure of the fiber cone of the anticanonical module, for a ring whose combinatorial structure connects to describe the anticanonical module and Frobenius complexity. In particular, I would like to study the description of the fiber cones of the anticanonical module using the ASL structure for rings with ASL structure.

On the other hand, Hibi ring is an Ehrhart ring of an order polytope, and Ehrhart rings of order polytopes and chain polytopes have many similarities. In particular, both are ASLs on the same distributive lattice, and I would like to study focusing on these similarities.