Research Plan

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1. The parallel transport map over affine symmetric space

As mentioned in "Research achievements", the parallel transport map is usually defined over a compact Riemannian symmetric space G/K. Since 2004, N. Koike has studied the parallel transport map over a symmetric space of non-compact type in the framework of pseudo-Riemannian submersion. Recently, I showed that, more generally, the parallel transport map is defined over an affine symmetric space without using a metric and it becomes an affine submersion with a horizontal distribution. (The paper is now in preparation.) Based on this fact, I will study the submanifold geometry of symmetric spaces via that of Hilbert spaces. Especially, I try to generalize the known results about affine submanifolds in Euclidean spaces to the case of Hilbert spaces, apply it to the PF submanifold $\Phi_K^{-1}(N) \subset V_{\mathfrak{g}}$ and study affine geometry of submanifolds in affine symmetric spaces.

2. Unique existence of minimal orbits in Hermann actions

For a given isometric action of a Lie group on a Riemannian manifold it is a fundamental problem to determine its minimal orbits. It is known that for the isotropy representation of a compact symmetric space G/K there exists a unique minimal orbit in each strata of the stratification of the orbit types (Hirohashi-Song-Takagi-Tasaki 2000). A similar property also holds for the isotropy action of G/K and more generally for commutative Hermann actions (Ikawa 2011). The purpose of this research is to extend this result to the case of Hermann actions which are not commutative. First I describe the orbit spaces of non-commutative Hermann actions in terms of root systems. If the theorem holds in the non-commutative case then I try to prove it. If it does not hold in that case then I will show a counterexample.

3. Isotropy representations of affine Kac-Moody symmetric spaces

Affine Kac-Moody symmetric spaces are infinite dimensional symmetric spaces proposed by C.-L. Terng and established by E. Heintze, B. Popescu and W. Freyn based on Kac-Moody theory. Many similar properties between those spaces and finite dimensional Riemannian symmetric spaces are known. In particular their isotropy representations are described by path group actions induced by Hermann actions or σ-actions. In this research I study the unique existence for minimal orbits in the isotropy representations of affine Kac-Moody symmetric spaces. In the finite dimensional case it is known that there exists a unique minimal orbit in each strata of the stratification of orbit types. I conjecture that the similar property also holds for affine Kac-Moody symmetric spaces. I aim to investigate and prove this conjecture.

4. Reformulation of integrable systems by affine Kac-Moody groups

It is known that the symmetries of soliton equations are described in terms of affine Kac-Moody algebras (M. Kashiwara, M. Jimbo, E. Date and T. Miwa 1980s). On the other hand, it is known that the transformations on solution spaces of some integrable equations can be described by loop group actions (C.-L. Terng and K. Uhlenbeck 2000s). An affine Kac-Moody group is the group corresponding to an affine Kac-Moody algebra and recently its realization is established in the framework of affine Kac-Moody symmetric spaces. An affine Kac-Moody group is realized as a T^2-bundle over a twisted loop group of smooth loops, which has a structure of tame Fréchet manifold (Hamilton 1982) and seems to be useful to study the relation between those two theories. I aim to make clear the relation between the above two methods for integrable systems through affine Kac-Moody groups.