Research Achievements

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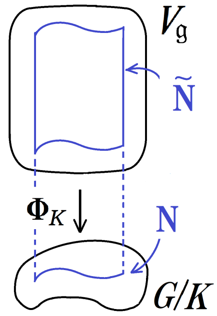
One way to study submanifolds in a compact Riemannian symmetric space G/K is to consider their lifts into a certain infinite dimensional Hilbert space. Let $V_{\mathfrak{g}} = L^2([0,1],\mathfrak{g})$ denote the Hilbert space of all L^2-paths from $[0,1]$ to the Lie algebra $\mathfrak{g}$ of G. C.-L. Terng and G. Thorbergsson studied a Riemannian submersion $\Phi_K : V_{\mathfrak{g}} \to G/K$, called the *parallel transport map*. For a closed submanifold N of G/K its inverse image $\tilde{N} := \Phi_K^{-1}(N)$ is a *proper Fredholm* submanifold of $V_{\mathfrak{g}}$, and its shape operators are compact self-adjoint operators. Although $\tilde{N}$ is infinite dimensional, many techniques in the finite dimensional Euclidean case are still valid due to linearity of the Hilbert space $V_{\mathfrak{g}}$. Using those techniques they studied submanifold geometry in symmetric spaces. It is a fundamental problem to show the geometrical relation between N and $\tilde{N}$.



In my research, I studied the geometric relation between N and $\tilde{N}$, especially the relation concerning the *symmetries* of minimal submanifolds.

In [1] I showed a relational formula between the shape operators of N and $\tilde{N}$. Then I showed a necessary and sufficient condition for $\tilde{N}$ to be a totally geodesic PF submanifold of $V_{\mathfrak{g}}$. Moreover I extended the concept of *weakly reflective* submanifolds (Ikawa-Sakai-Tasaki) to the class of PF submanifolds in Hilbert spaces. Then I showed that each fiber of Φ_K is a weakly reflective PF submanifold of $V_{\mathfrak{g}}$. Moreover I showed that if N is a weakly reflective submanifold of G/K then $\tilde{N}$ is a weakly reflective PF submanifold of $V_{\mathfrak{g}}$. Using these results I showed many examples of infinite dimensional weakly reflective PF submanifold of $V_{\mathfrak{g}}$ which are not totally geodesic.

In [2], using the formula for the shape operator obtained in [1], I showed a relational formula for the principal curvatures of N and $\tilde{N}$ under the assumption that N is a curvature-adapted submanifold. This gives another proof of the formula of N. Koike. Next, using this relational formula I studied the relation between the *austere* properties of N and $\tilde{N}$. By definition we have the relation "weakly reflective $\Rightarrow$ austere $\Rightarrow$ minimal". Although the principal curvatures of $\tilde{N}$ are complicated in general, I supposed that G/K is the standard sphere and showed that N is austere if and only of $\tilde{N}$ is austere. Moreover I studied the relation between the *arid* properties of N and $\tilde{N}$. Here arid submanifolds were introduced by Y. Taketomi and they generalize weakly reflective submanifolds. I showed that if N is arid then $\tilde{N}$ is also arid. Using those results I gave examples of arid PF submanifolds which are not austere.

In [3] I extended the results of [1] to the case that G/K is a compact isotropy irreducible Riemannian homogeneous space.

In [4], as an extension of [2], I studied the relation between the austere properties of N and $\tilde{N}$ under the assumption that N is an orbit of a *Hermann action*. First I introduced a hierarchy of curvature-adapted submanifolds in G/K and refined the formula for the principal curvatures given in [3]. Using this formula I derived an explicit formula for the principal curvatures of $\tilde{N}$ under the assumption that N is an orbit of a Hermann action. This formula generalizes some results by C.-L. Terng, U. Pinkall and G. Thorbergsson. Using this formula I showed that if N is austere then $\tilde{N}$ is also austere and that the converse is not true by showing a counterexample.

In [5] I introduced a canonical isomorphism of path spaces and extended the results of [4] to the case of σ-*actions*. Furthermore I made clear the relation among the known computational results of principal curvatures of PF submanifolds in Hilbert spaces. Furthermore, in Misc [13], I showed that the isomorphism given in the paper is induced by a natural isomorphism of infinite dimensional symmetric spaces, called affine Kac-Moody symmetric spaces.

In [6] (preprint), I showed that polar actions on Hilbert spaces by connected Lie groups have no exceptional orbits. Using this result I gave a simple geometric proof of the fact that hyperpolar actions on G/K by connected Lie groups have no exceptional orbit.