Research Plan

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Integrability of cohomogeneity one Nambu-Goto string:

The motion of point particles in spacetime is described by geodesics. The geodesic equations are equivalent to a dynamical system with the spacetime metric as its Hamiltonian, and become integrable when there exist a sufficient number of commuting conserved quantities. When spacetime possesses symmetries, there exist Killing vector fields corresponding to these symmetries, and if the number of conserved quantities constructed from these Killing vectors is sufficient, the geodesics in that spacetime become integrable. Interestingly, even in spacetimes like the Kerr spacetime where symmetries are limited and the number of Killing vectors is insufficient, geodesics can still be integrable. Such spacetimes are said to possess 'hidden symmetries,' where the existence of Killing tensors guarantees new conserved quantities and ensures the integrability of the geodesic equations.

Extending the concept of 'hidden symmetries' associated with point particle (zero-dimensional object) motion to explore the possibility of 'hidden symmetries' associated with string (one-dimensional object) motion represents an important research direction. The Nambu-Goto string, which takes the area of the string's world-sheet as its action, is known as a natural generalization of the geodesic equation. However, the requirement for integrability of general Nambu-Goto strings imposes excessively strong constraints on spacetime, resulting in limited examples of such spacetimes.

This research focuses on cohomogeneity-one Nambu-Goto strings (strings aligned with Killing vector fields in only one direction of the world-sheet) and aims to characterize spacetimes that satisfy their integrability conditions. While it is known that all cohomogeneity-one strings are integrable in maximally symmetric spacetimes, there exist examples in sub-maximally symmetric spacetimes where this is not the case. This research seeks to establish necessary and sufficient conditions for the integrability of all cohomogeneity-one strings and to identify the boundary between integrability and non-integrability. Furthermore, as a natural extension of this research, we will investigate the possibility of extending these results to cohomogeneity-one membranes.

Loop quantum gravity and complexity:

Loop quantum gravity (LQG) is an approach to non-perturbative and background independent quantization of general relativity. Spin-network states span the kinematic state space of LQG as an orthonormal basis. Spin-network state is labeled with spin-network, which is a graph whose edges are colored by half integers satisfying simple relations at the vertices. In LQG, Geometrical operators (area operator and volume operator) are constructed. Their eigenstates are spin-network states and they have discrete eigenvalues.

From informational viewpoint on volume operator in LQG, following conjecture is suggested: A logic gate must have finite minimum volume, and number of logic gates contained within a region of space are bounded by the volume of the region.

It seems to say "Complexity is Volume."

We want to consider the possibility of LQG as a foundation where complexity is discussed.