Summary of research results

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When considering classical field theory, solutions where energy is localized within a finite region due to nonlinear effects of the field can emerge. These are called soliton solutions. Solitons are broadly categorized into two types. The first is "topological solitons," also known as topological defects, and the second is "non-topological solitons," whose stability is characterized by conserved quantities associated with the symmetry of the theory. Both types of solitons have been explored for applications in astrophysics.

The applicant has focused on constructing various soliton solutions using numerical computations with Python and Mathematica. These studies are based on a theoretical model described by the Lagrangian

$$\mathcal{L} = -g^{\mu\nu}(D_\mu\phi)^*(D_\nu\phi) - g^{\mu\nu}(D_\mu\psi)^*(D_\nu\psi) - \frac{\lambda}{4}(|\phi|^2 - \eta^2)^2 - \mu\,|\phi|^2\,|\psi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1}$$

and have explored their applications. The proposed model by the applicant follows these steps:

1. Introduction of a complex scalar field : Consider a theory with a complex scalar field ψ.
2. Extention by gauge field : Introduce a U(1) gauge field A_μ to extend the theory.
3. Spontaneous symmetry breaking: Introduce a complex Higgs field ϕ that causes the spontaneous symmetry breaking.

The construction naturally fits within the framework of gauge theories, leading to a well-motivated theoretical extension. Despite its natural theoretical structure, the model exhibits the emergence of unique solutions, such as non-topological soliton solutions, which the applicant has successfully demonstrated.

In the proposed model, the applicant conducted an analysis focusing on spherically symmetric non-topological solitons, referred to as Q-balls, with particular attention to astrophysically significant Q-balls with enormous masses. The study demonstrated that the model supports three distinct types of characteristic solutions, classified based on the equation of state inside the Q-ball:

- Dust Ball : The energy inside the Q-ball is finite, but the pressure approaches zero.
- Shell Ball : Both energy and pressure inside the Q-ball vanish, forming a shell-like solution.
- Potential Ball : The ratio of energy to pressure inside the Q-ball equals -1.

Among these solutions, we demonstrated that Dust Balls and Shell Balls can possess infinitely large masses.

Furthermore, the applicant constructed solutions called boson stars by coupling the model to the gravitational field. This introduced a new parameter, $\sqrt{G}\eta$, defined as the ratio between the Planck energy and the symmetry-breaking energy scale η. In the limit where $\sqrt{G}\eta \to 0$, the equations reduce to those of Q-balls. Therefore, at sufficiently small energy scales η, boson star solutions similar to Q-ball solutions are expected to emerge. However, even a slight but non-zero value of $\sqrt{G}\eta$ necessarily imposes an upper mass limit to prevent gravitational collapse.

The applicant first conducted an analysis based on Dust Ball-type boson star solutions. The study revealed that at a critical value of $\sqrt{G}\eta$, the behavior of the fields and physical quantities of the boson star undergo significant changes. The analysis led to the following conclusions regarding Dust Ball-based boson stars:

- They can attain a mass comparable to that of supermassive black holes at galactic centers.
- While photon spheres do not exist, ISCOs (Innermost Stable Circular Orbits) can form.

In the framework of relativity, a hypothetical celestial object known as the "gravastar" has been proposed as one of the possible black hole alternatives. Gravastar solutions are generally constructed by artificially connecting the internal and external spacetime structures using Israel's junction conditions.

The type of Q-ball solution that the applicant refers to as a 'Potential Ball' has the distinctive feature of being filled with vacuum energy within the soliton interior. When considering a boson star based on this Potential Ball, it becomes possible to construct a boson star filled with vacuum energy in its interior. The study revealed that this configuration shares properties similar to those of gravastars.

Furthermore, the analysis showed that this "gravastar-like boson star" can achieve a density high enough to form photon spheres. This characteristic sets it apart as a unique and theoretically significant black hole alternative.