# （2） Future Research Plan

Soma Ohno

## Backgrounds to Research Plans

As already mentioned in the "Summary of Previous Research", it is important to investigate Rarita-Schwinger fields on manifolds with real Killing spinors. Manifolds with real Killing spinors are classified by C. Bär as either Sasaki-Einstein manifolds, 3-Sasakian manifolds, nearly Kähler manifolds or nearly parallel $G_2$-manifolds. In peer-reviewed papers [1] and [2], Rarita-Schwinger fields on nearly Kähler manifolds and nearly parallel $G_2$-manifolds have been clarified, respectively. Investigating Rarita-Schwinger fields on the remaining Sasaki-Einstein manifolds and 3-Sasakian manifolds is very interesting, since I expect that Rarita-Schwinger fields have applications to deformation theory on the respective manifolds.

## Research Plans

**(1) Study Rarita-Schwinger fields on Sasaki-Einstein manifolds**

Recently, U. Semmelmann, C. Wang, and M.-Y. Wang obtained the result about the linear stability of Sasaki-Einstein manifolds. Linear stability of Einstein metrics is the notion defined as the second variation of the Riemann functional, called the total scalar curvature, in the TT-direction being negative. Now, their result is that Sasaki-Einstein manifolds are linearly unstable if the Betti number satisfies certain conditions. The method is to check linear instability by calculating the difference between the Laplacian for a "good connection" on Sasakian manifolds and the Laplacian for the Levi-Civita connection, and rewriting the harmonic form. I believe that this method can be applied to investigate Rarita-Schwinger fields on Sasaki-Einstein manifolds. I specifically plan to research the following.

There is a special vector field $\xi$ called the Reeb vector field on $(2n+1)$-dimensional Sasaki-Einstein manifolds. The tangent bundle decomposes as $TM = D \oplus \langle \xi \rangle$, where $D$ is the $2n$-dimensional normal bundle of $\langle \xi \rangle$ and is equipped with a transversal Kähler structure. Furthermore, it is known that the spinor bundle $S_{1/2}$ decomposes into a sum of certain irreducible vector bundles by using this transversal Kähler structure and real Killing spinors. Thus the spin-3/2 spinor bundle $S_{3/2} \subset S_{1/2} \otimes TM$ is decomposed into a sum of certain vector bundles.

There exists a "good connection" on Sasaki-Einstein manifolds. I will construct formulas between the twisted Dirac operator and the Rarita-Schwinger operator for this "good connection" and the Levi-Civita connection. Using these tools, I rewrite Rarita-Schwinger fields into tensor products. In fact, I am applying this method to 5-dimensional Sasaki-Einstein manifolds and am working hard on the calculations.

**(2) Study Rarita-Schwinger fields on 3-Sasakian manifolds**

3-Sasakian manifolds are a special case of Sasaki-Einstein manifolds. I therefore expect to be able to develop the discussion in (1). I will also stay at the University of Stuttgart, Germany, for three weeks in February 2025 to discuss these studies with U. Semmelmann.

**(3) Study invariant Rarita-Schwinger fields on nilpotent manifolds**

Recently, G. Bazzoni, L. Martin-Merchan, and V. Munoz studied eigenvalues of harmonic spinors and Dirac operators on nilpotent manifolds. They have taken the approach that Dirac operators acting on invariant spinors can be easily expressed and computed separately for concrete nilpotent manifolds. We believe that this method can be applied to RS operators as well. Furthermore, in the future, we expect to be able to study invariant Rarita-Schwinger fields on various Lie groups.