

## Plan of Research

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### Properties of constituent split links of a handlebody-knot

We have shown the uniqueness of the Alexander polynomial of constituent split links of a handlebody-knot. A natural question is whether the group of constituent split links of a handlebody-knot is uniquely determined, or whether the constituent split link of a handlebody-knot is uniquely determined.

If the constituent split link  $L = K_1 \cup K_2$  of a handlebody-knot is uniquely determined, then is the handcuff graph  $\Gamma = L \cup c = K_1 \cup K_2 \cup c$  of a handlebody-knot uniquely determined? If the handcuff graph  $\Gamma$  is uniquely determined, then we consider  $\Gamma$  as the standard form of a handlebody-knot with a constituent split link, and the problem of classification of handlebody-knots is reduced to classification of  $\Gamma$ .

If the constituent split link of a handlebody-knot is not uniquely determined, then the set of all constituent split links is an invariant of the handlebody-knot. Among the constituent links of a handlebody-knot, the constituent split link is easy to calculate, so we would like to investigate how strong an invariant it is.

### A lower bound for the crossing number of a handlebody-knot

We have a lower bound for the crossing number of constituent links of a handlebody-knot. To prove this, we use a property of a C-complex of a constituent link of a handlebody-knot. We introduced a C-complex of a handlebody-knot. I would like to consider a lower bound for crossing number of a handlebody-knot as an analogy.

### Alexander ideal of a handlebody-knot

We introduced the graph  $G_H$  as an invariant of handlebody-knot  $H$  derived from the Alexander polynomial. There is infinite many handlebody-knots whose Alexander polynomial is trivial and whose Alexander ideal is non-trivial. I would like to expand the invariant  $G_H$  by using the Alexander ideal of a handlebody-knot.