Research Plan

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1 Normal Form Theory of Second-Order Equations

As is explained in Research Achievements, by applying the method to prove Borel summability of transformation to the canonical equation near a double turning point, Borel summability of various kinds of transformations is shown([1]). By this result, normal form theory of second-order equations is almost complete, but there are some exceptional problems like transformation near a so-called ghost point, transformation near a triplet of simple turning points, etc. Such problems will be studied.

2 Analytic Foundation of the Exact WKB Analysis of Painlevè Equations

The exact WKB analysis of Painlevè equations was vigorously studied by Aoki, Kawai, Takei, et. al., and many interesting results are known. However there are formal solutions and formal transformations in the theory, for which we need to give analytic foundation.

As for formal transformations, due to development in the transformation theory of linear equations, we have an outlook to establish Borel summability, and I attack this problem. As for formal solutions, summability of 0-parameter solutions (formal power series solutions) and 1-parameter solutions (transseries solutions) is known, but analytic interpretation of 2-parameter solutions is still open. About this problem, the simplest equation, i.e., the first Painlevè equation is studied and summability is being obtained, and I try to solve the problem generally. Furthermore, we aim to connect the resummed solutions and asymptotically elliptic behavior of solutions, which is classically known.

In the case of higher-order painlevè equation, summability of formal solutions is more difficult than that of higher-order linear equations, but transformation theory for some equations can be handled in a similar manner to the transformation theory of second-order linear equations. As for transformation theory, we also aim to deal with such higher-order Painlevè equations.

3 Borel Summability of WKB Solutions and Normal Form Theory in Higher-Order Linear Equations

The exact WKB analysis of higher-order equations has recently been used in geometry and mathematical physics, and to solve fundamental problems such as Borel summability of WKB solutions is more desired. I myself had researched toward Borel summability of WKB solutions of higher-order equation, as my main research theme for some years.

However this is an extremely difficult problem which has been unsolved for about 40 years since its importance and difficulty was recognized, and I could not make a significant progress, too. Therefore I will treat this problem only as one my subthemes for some time, continue analysis of concrete examples by the exact steepest descent method etc. to gain our knowledge, and aim to solve it in the future. Also, beyond this problem, I look ahead normal form theory (especially Borel summability of transformation) of higher-order equations. This is also a natural succeeding research of my results dealing with Stokes phenomena in higher-order equations, bifurcation at a double turning point etc.