Future research plans

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I am planning to investigate, from a geometrical view point, how the symmetry appears in mathematics as I partially revealed it so far. Concretely, I will attack the following problems.

Problem 1. The parallelogram in Figure 1 (of another sheet) is a realization of the breaking symmetry in both of geometry and theory of symmetric functions. What are the corresponding objects in the case of other Lie types?

Problem 2. GKM theory (explained in detail later) seems to be the most efficiently applied to flag varieties and other subvarieties (or submanifolds). How can the connection between Hessenberg varieties and their twins be interpreted in GKM theory?

Problem 3. What are corresponding geometrical objects to non-unicellular LLT polynomials?

First I explain GKM theory. The equivariant cohomology ring (the ordinary cohomology ring of the Borel construction) of a space with a good torus action is determined by the fixed point set and its one dimensional orbit. This is the main theorem of GKM theory. The construction of the twin from a Hessenberg variety closely relates to the Borel construction, and the twin itself has a good torus action. Hessenberg varieties and their twins seem to be deeply connected through GKM theory.

The followings are my concrete aims for these problems in a few years.

Aim 1. Find them in the case of type C. Towards this aim, construct the twins of regular semisimple Hessenberg varieties of type C, and then calculate the cohomologies of both of them. Detect corresponding symmetric functions and find combinatorial descriptions of them.

Aim 2. Develop GKM theory in the sense of the duality of tori which act on Hessenberg varieties and their twins. Especially, develop it to interpret the interchanging phenomenon of the fiber and the base space of fiber bundles associated with those spaces.

Aim 3. Give a generalization of the twins corresponding to vertical-strip LLT polynomials.

These problems relate to each other, and then partial achievements I will obtain can be applied to the other problems (see Figure 2). For development of GKM theory, detect a class of appropriate spaces. And specific examples can be obtained from the other problems. Geometrical understanding of development of GKM theory give us a clue to the twins. Especially, one can make calculations very concretely in GKM theory.

Figure 2: Connections between aims