A summary of past research achievements

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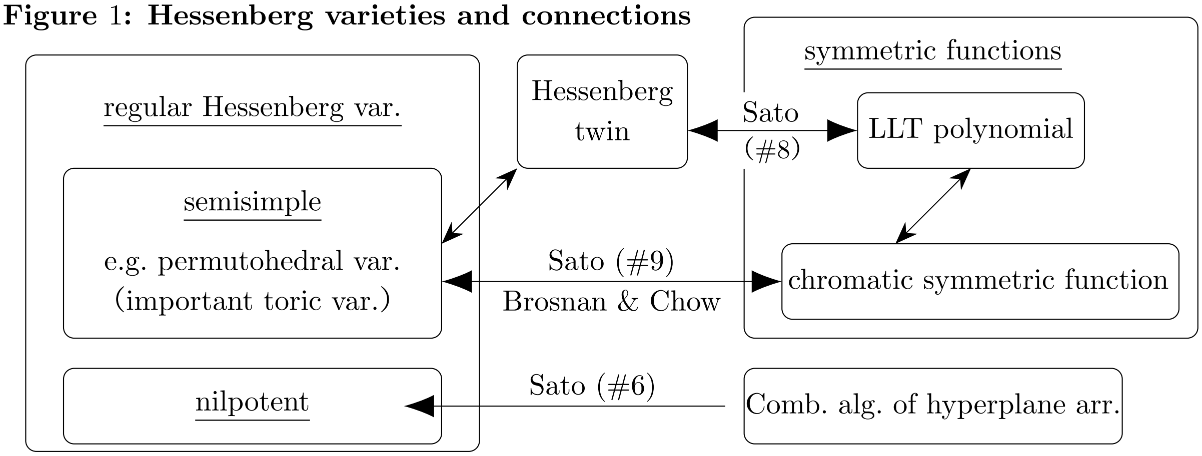
(1) Research Objects

Flag varieties are important because they geometrically reflects the symmetries of their Weyl groups and root systems. **Hessenberg varieties** are “good” subvarieties of a flag variety, which are obtained by breaking its symmetry in some sense. Hessenberg varieties give us geometrical ways to investigate the symmetry from the outside perspective of the symmetry. Recently, I am studying Hessenberg varieties and have revealed connections among geometry, combinatorics, and representation theory. (See horizontal arrows in Figure 1.)

Figure 1: Hessenberg varieties and connections



(2) Research achievements

First I explain my most important result, namely the parallelogram in Figure 1. Assume that Hessenberg varieties are regular semisimple and of type A (Later I refer this condition by ($*$).) The cohomologies of such Hessenberg varieties has a symmetric group action, and then they are graded representations of the symmetric group. Brosnan and Chow proved that the representation coincides with an important symmetric function, the chromatic symmetric function (csf for short) of a corresponding graph (with an involution). I obtained an alternative simple proof of this fact. (See paper #9 of the paper list.)

There is another important class of symmetric functions. They are LLT polynomials, which correspond to the graphs above. To explain my result I need another object, the “twins” of Hessenberg varieties of ($*$). I revealed that the twin is to a Hessenberg variety what a unicellular LLT polynomial is to a csf corresponding to the same graph. (Hence I illustrate their connections as a parallelogram in Figure 1. See paper #8.) As a consequence, I proved that the cohomology of the twin coincides with the corresponding LLT polynomial.

I determined when Hessenberg varieties of ($*$) satisfy the condition where their cohomology rings are generated by degree two elements. (See papers #7 and #11.)

I described the cohomology rings of regular nilpotent Hessenberg varieties in terms of some combinatorial algebraic objects corresponding to hyperplane arrangements of the breaking symmetry. (See paper #6.) This is an generalization of the result of Borel for flag varieties.

I determined an explicit description of the equivariant cohomology rings of flag varieties of type C, F_4, and E_6 as generators and relations. (See papers #2, #3, and #10)

In the above researches, GKM theory was used. It is a theory for spaces with a good torus action, and it provides us to a way to compute their (equivariant) cohomologies from corresponding graphs with labels. When the space has an almost complex structure, I showed that its automorphism group is a subgroup of the automorphism group of the labeled graph. (See paper #12)