Research Plan

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KP and Toda hierarchies of B, C and D types Soon after the KP hierarchy was devised as a universal integrable system in the early 1980's by M. Sato et al., variations of the KP hierarchy of the B and C types (the BKP and CKP hierarchies) were introduced from the point of view of infinite dimensional Lie algebras. Similar variations of the Toda hierarchy were also sought for when it was introduced by K. Ueno and myself a few years later. The D type (the DKP hierarchy) and a new version of the B type (the large BKP hierarchy) of the KP hierarchy were also discovered in later studies. Recently, new variations of the Toda hierarchy (the B-Toda and C-Toda hierarchies) were proposed by A. Zabrodin et al. I would like to elucidate their detailed properties, in particular, infinite dimensiomal symmetries.

Riemann-Hilbert problem in supersymmetric gauge theories and topological string theories T. Bridgeland introduced a Riemann-Hilbert problem in geometry of stability conditions, and presented several examples where the problem can be solved by the Gamma and multiple-sine functions. M. Alim et al. considered these examples from the point of view of topological string theory. Bridgeland's Riemann-Hilbert problem originates from the work of D. Gaiotto, G. Moore and A. Neitzke on a kind of hyper-Kähler metrics. Those metrics are constructed for the moduli spaces of vacua in supersymmetric gauge theories. I would like to study in more detail the roles that the Riemann-Hilbert problem plays in mathematical physics.

Enumerative geometry and Frobenius manifolds B. Dubrovin formulated the geometric structure of genus-zero Gromov-Witten theory as a Frobenius manifold, and pointed out its relation to integrable systems and isomonodromic deformations. A. Givental presented an expression for the generating functions of all-genus Gromov-Witten invariants in terms of operators on a bosonic Fock space. According to Givental, this operator representation can be thought of as quantization of a Lagrangian submanifold in infinite-dimensional symplectic geometry. I would like to understand its meaning and its impact on integrable systems.

Grassmann manifolds and boundary measurement A. Postnikov constructed a cellular decomposition of the nonnegative part of a real Grassmann manifold employing the notion of boundary measurement. The boundary measurement is a mapping that sends a weighted graph to a point of the Grassmann manifold by a sum over paths or matchings on the graph. Postnikov also pointed out that diverse combinatorial notions underlie the cellular decomposition. Since Postnikov's work, the boundary measurement and the related combinatorial notions have been applied to the scattering amplitudes of four-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory. Several different approaches to the scattering amplitudes are known, and I would like to clarify their relations.

History of mathematics Over the past few years, I have looked through 19th century texts on topics such as the matrix-tree theorem, the origin of Grassmannian manifolds, and the interlace theorem for eigenvalues of real symmetric matrices, and have found some interesting facts. My recent interest lies in Sturm's theorem on the number of roots of polynomials, and I would like to investigate the related literature.