

Research Plan

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Classification of 2-plat 2-knots

An important result related to a plat form representation of knots is the complete classification of 2-bridge knots. This classification is not only useful as a computational example in knot theory but also yields interesting results regarding geometric structures. As a higher-dimensional generalization, I aim to tackle the classification of 2-plat 2-knots. I have already obtained specific results, such as proving that 2-plat 2-knots under certain conditions possess the property of being ribbon and deriving explicit formulas for their Alexander polynomials. Moving forward, I intend to complete the classification of 2-plat 2-knots by determining the structure of their branched covers and comparing various invariants.

Construction of new invariants for surface-knots using quandles

A quandle is an algebraic system obtained by axiomatizing the equivalence transformations of knots algebraically. From the perspective of group theory, it can be interpreted as an algebraic system generalizing conjugation. In particular, for knots, a quandle called the knot quandle can be defined, which serves as a powerful invariant capable of almost completely determining the equivalence class of a knot.

For orientable surface-knots, the knot quandle can also be introduced. Currently, the knot quandle is actively discussed in the study of surface-knots. On the other hand, for general surface-knots, including non-orientable cases, it is necessary to consider a quandle with an additional structure called a symmetric quandle. However, many aspects of this symmetric quandle remain unclear. I have constructed an algorithm that provides a representation of the knot symmetric quandle (the symmetric quandle version of the knot quandle) from plat form representation of surface-links. Furthermore, I have provided necessary and sufficient conditions for a given symmetric quandle to be the knot symmetric quandle of a surface-link. In particular, symmetric quandle homology, defined for symmetric quandles, is expected to play a crucial role in constructing invariants for surface-links and understanding symmetric quandles. Therefore, I aim to further deepen the understanding of this theory.